DEVELOPMENT AND STUDY OF A TWO-DIMENSIONAL DYNAMIC MODEL OF A TOWED BUOY

by

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THESIS

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ABSTRACT

A two-dimensional dynamic model of a towed buoy designed to remain submerged just below the water surface is developed. The study includes the model development, the analysis of certain model parameters, the development of a tension force, and the range of stability of the model to initial condition perturbations.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

TABLE OF SYMBOLS AND ABBREVIATIONS

F	force, in pounds
m	mass, in slugs
a	acceleration, in ft/sec^2
M	moment, in ft·pounds
I_y	mass moment of inertia about Y axis, in slug·ft^2
$\ddot{\theta}$	angular acceleration in rad/sec^2
D	drag, in pounds
C_L	coefficient of lift
C_D	coefficient of drag
C_{DA}	coefficient of drag of the antenna
C_M	coefficient of the pitching moment about the towpoint
ρ	mass density of water, approximately equal to 1.99
S	characteristic area of the buoy
$\bar{C}$	mean chord length of the buoy
b	span of the buoy
T	tension, in pounds
L	lift, in pounds

ACKNOWLEDGEMENT

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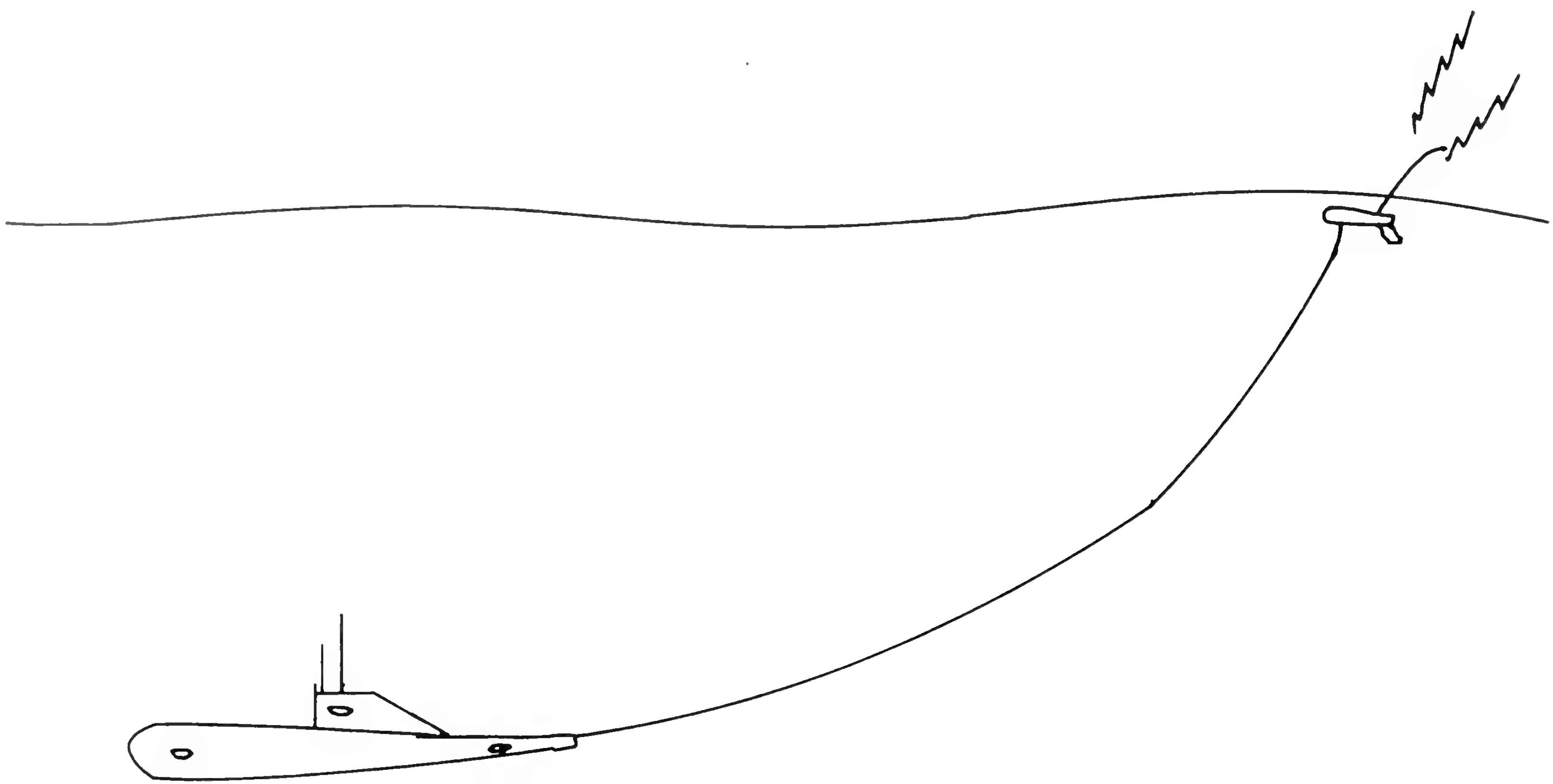
I. INTRODUCTION

The purpose of this investigation is to attempt to mathematically model and computer-simulate the two-dimensional motion of a submarine-towed buoy. This has a very specific military application in submarine communications systems; the basic system configuration as a submarine communication link is illustrated in Fig. 1. It is desired that the buoy stay underwater at all times, and remain within ± 1 ft. of four feet below the water surface; this gives the buoy an operating range from three to five feet below the surface. An additional system requirement is that the buoy system stay within its allowable operating zone while following waves up to sea-state four.

Many investigators, such as Lacey, have dealt with ship-towed or airplane-towed devices, where the buoy is the "bottom-following" type. Pode and other early investigators studied the steady-state configuration of cables in a steady uniform stream; and more recently Schram developed, by the method of characteristics, a two-dimensional dynamic cable-buoy model. However, he treated the buoy as a vector force and did not go into the actual buoy dynamics.

Steady-state studies do not show the response of the buoy and cable to perturbations such as sea state. It is this response that will prove most useful in the study of buoy and cable parameters. This study has been primarily concerned with developing the dynamic model of the buoy; and the tension force generated by the cable has been loosely approximated.



SUBMARINE COMMUNICATIONS BUOY

Figure 1

The model is of a buoy presently being developed for the U. S. Navy; thus, the dimensions and other data were obtained from an existing prototype.

This study develops the two-dimensional model around two basic equations; force equals mass times acceleration, and the moment equals the mass moment of inertia times the angular acceleration. The model block diagram was built around these two equations, several basic hydrodynamic relationships, and the wind-tunnel and dimensional data supplied on the buoy. Once the block diagram was formulated, it was then ready to be computerized. The model was simulated using the IBM 360-67 at the Naval Postgraduate School, and DSL (Digital Simulation Language); DSL is a very simple, but versatile simulation language developed by IBM. Once the model was computerized, the lengthy and time-consuming task of optimizing certain parameters and finding a realistic formulation of the tension force followed. The determination of the tension force and its coupling with the buoy has been one of the most difficult problems in the formulation of the buoy-cable model. This has been a problem in all buoy-cable models where the dynamic response is of interest; this is mainly due to the fact that it is not exactly known how the forces generated by the buoy and cable react on and with each other. The development of the model and the results of the simulation are presented.

II. EQUATIONS OF MOTION AND PROBLEM SETUP

The purpose of the buoy is to remain four feet below the water surface so that a protruding antenna may maintain two-way communications; when the buoy is operating in high seas it must follow the sea surface. The body, as shown in Fig. 2., is basically a fat wing that produces dynamic lift according to its angle of attack. This angle of attack may be changed by torque produced by the fin. Besides the dynamic lift, produced by the body angle of attack, there is the static lift of positive buoyancy. The buoy model is based on the two basic equations:

$$F=ma$$

$$M=I_y\ddot{\theta}$$

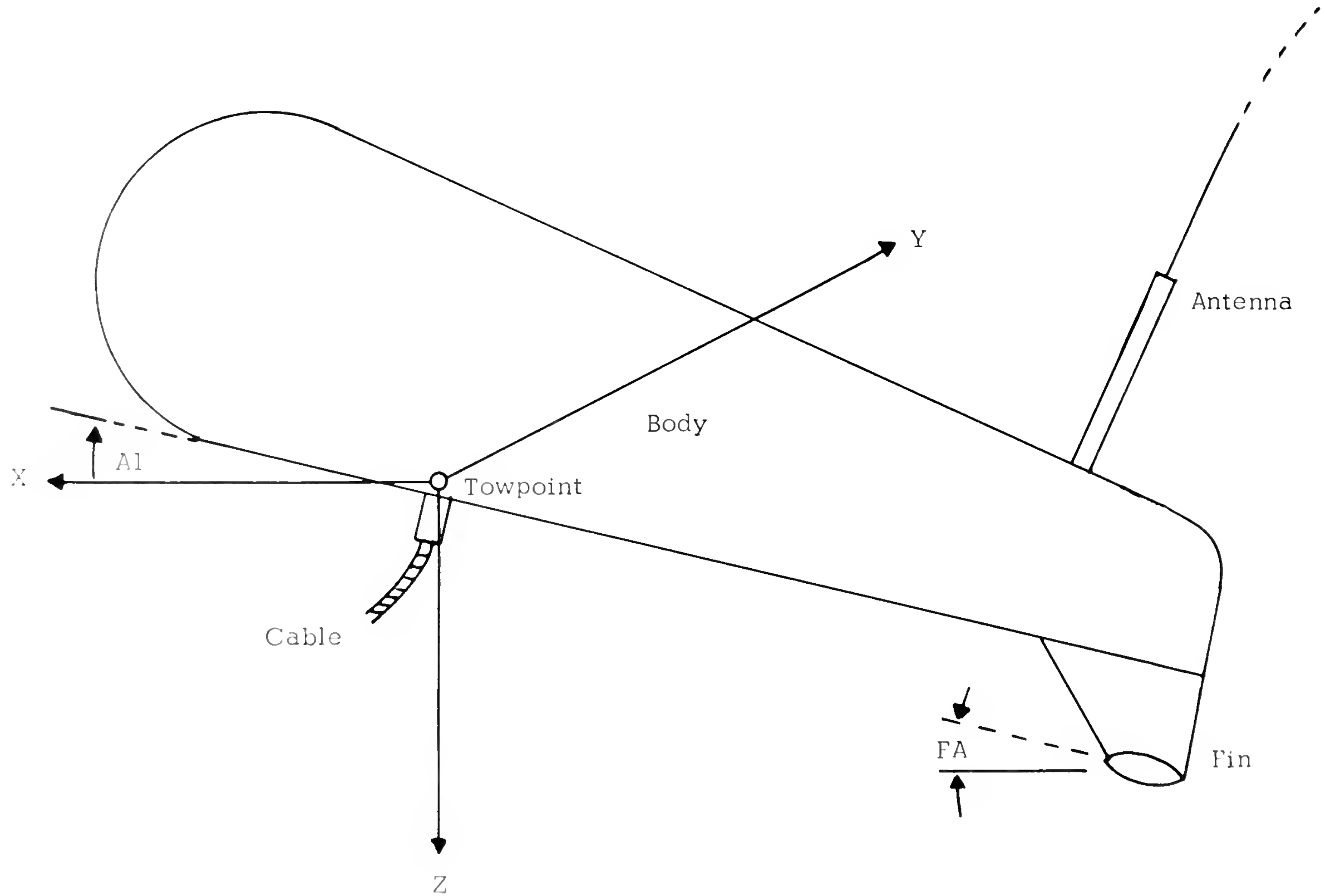
The first equation is used to describe movement in the horizontal and vertical planes, the X and Z planes respectively. The second is used to describe the rotation of the body about the Y axis; these axis are shown in Fig. 2. The three hydrodynamic forces of lift, drag, and moment are defined as follows:

$$L=\tfrac{1}{2}C_L\rho V^2 S$$

$$D=\tfrac{1}{2}C_D\rho V^2 S$$

$$M=\tfrac{1}{2}C_M\rho V^2 S\overline{C}$$

Along with the tension force produced by the cable and the buoyancy of the body, these three equations describe all the forces acting on the



THE BUOY

Figure 2

body. Before going into the complete buoy block diagram, the body coefficients will be put into a more convenient form, and the fin servo loop, which controls the buoy attitude, will be examined.

A. BODY COEFFICIENTS

In both aerodynamics and hydrodynamics, there are various dynamic coefficients; these are determined by the shape of the body involved. There are tables of coefficients for certain common wing shapes. However, when an odd body configuration is being studied, it may not be possible to apply the standard shapes in order to derive overall coefficients for the body. In this case the coefficients of the body must be determined by wind-tunnel tests.

This was the case with the system being studied; the body is of non-standard shape, but the fin is the standard NACA 0012 shape. The three coefficients, C_L, C_D, and C_M, were measured at various body and fin angles. The coefficient of lift and the coefficient of moment include the effect of the positive buoyancy force of 490 pounds. The effect of the antenna drag is not included in any of the coefficients. The values of these coefficients are shown in Tables I-III as a function of body angle of attack and fin angle of attack. It was decided that for computer implementation, these coefficients would be easier to handle if they were in equation form. This was done by applying a least-squares fit twice to each data set; the resulting equations may be found in Subroutine Tink of the digital program of the model. The resultant curves and

FIN ANGLE (Deg.)

BODY ANGLE (Deg.)	-10	-7.5	-5.	-2.5	0.0	2.5	5.
-12	-.054	-.039	-.027	-.016	-.003	.0064	.0211
-9	-.001	.0110	.0253	.0353	.0477	.0589	.0756
-6	.0585	.0647	.0776	.0901	.1020	.1139	.1306
-3	.1102	.1214	.1351	.1461	.1593	.1723	.1876
0	.1676	.1792	.1929	.2035	.2166	.2294	.2454
3	.2315	.2432	.2573	.2677	.2807	.2943	.3095
6	.3044	.3163	.3285	.3414	.3547	.3678	.3828
9	.3834	.3946	.4075	.4195	.4333	.4461	.4593
12	.4654	.4774	.4911	.5025	.5155	.5298	.5386

COEFFICIENT OF LIFT

Table I

BODY ANGLE (Deg.)	FIN ANGLE (Deg.) -10	-7.5	-5.	-2.5	0.0	2.5	5.
-12	.0415	.0374	.0364	.0331	.0327	.0307	.0320
-9	.0345	.0314	.0310	.0284	.0283	.0271	.0288
-6	.0319	.0302	.0298	.0279	.0287	.0284	.0306
-3	.0347	.0334	.0336	.0322	.0343	.0347	.0377
0	.0403	.0404	.0415	.0404	.0430	.0444	.0475
3	.0514	.0522	.0537	.0534	.0568	.0583	.0625
6	.0696	.0703	.0721	.0730	.0769	.0804	.0844
9	.0956	.0972	.0998	.1007	.1048	.1087	.1143
12	.1306	.1326	.1354	.1377	.1428	.1475	.1522

COEFFICIENT OF DRAG

Table II

FIN ANGLE (Deg.)

		-10	-7.5	-5.	-2.5	0.0	2.5	5.
	-6	.0981	.0908	.0834	.0772	.0707	.0639	.0554
	-3	.0794	.0723	.0651	.0593	.0528	.0452	.0370
	0	.0585	.0512	.0439	.0382	.0314	.0237	.0158
BODY ANGLE (Deg.)	3	.0539	.0289	.0214	.0156	.0086	.0010	-.007
	6	.0106	.0036	-.003	-.009	-.017	-.024	-.032
	9	-.016	-.023	-.030	-.036	-.044	-.051	-.058
	12	-.044	-.050	-.057	-.064	-.071	-.078	-.-84

COEFFICIENT OF MOMENT

Table III

data points for constant body angle of attack and varying fin angle of attack are shown in Figs. 3-5.

B. FIN SERVO LOOP

The diagram of the fin and the servo-control system is shown in Fig. 6. The purpose of the fin is to orient the body so that it will stay four feet below the water surface. The fine produces a moment on the buoy which causes the body angle of attack to increase or decrease. This increase or decrease in body angle of attack causes the body lift to increase or decrease and the body rises or descends as is necessary.

The fin actuator loop is driven by a Size 23, Kearfott motor. The loop is designed to have a rise time of 0.04 sec.; this actuator loop is shown in Fig. 7. The servo loop used in the buoy model is considered to be a K/s^2 plant with tachometer and unity feedback; this is shown in Fig. 8. The system transfer function:

$$\frac{G}{1 + GH} = \frac{K_A/s^2}{1 + K_A/s^2(K_t s + 1)}$$

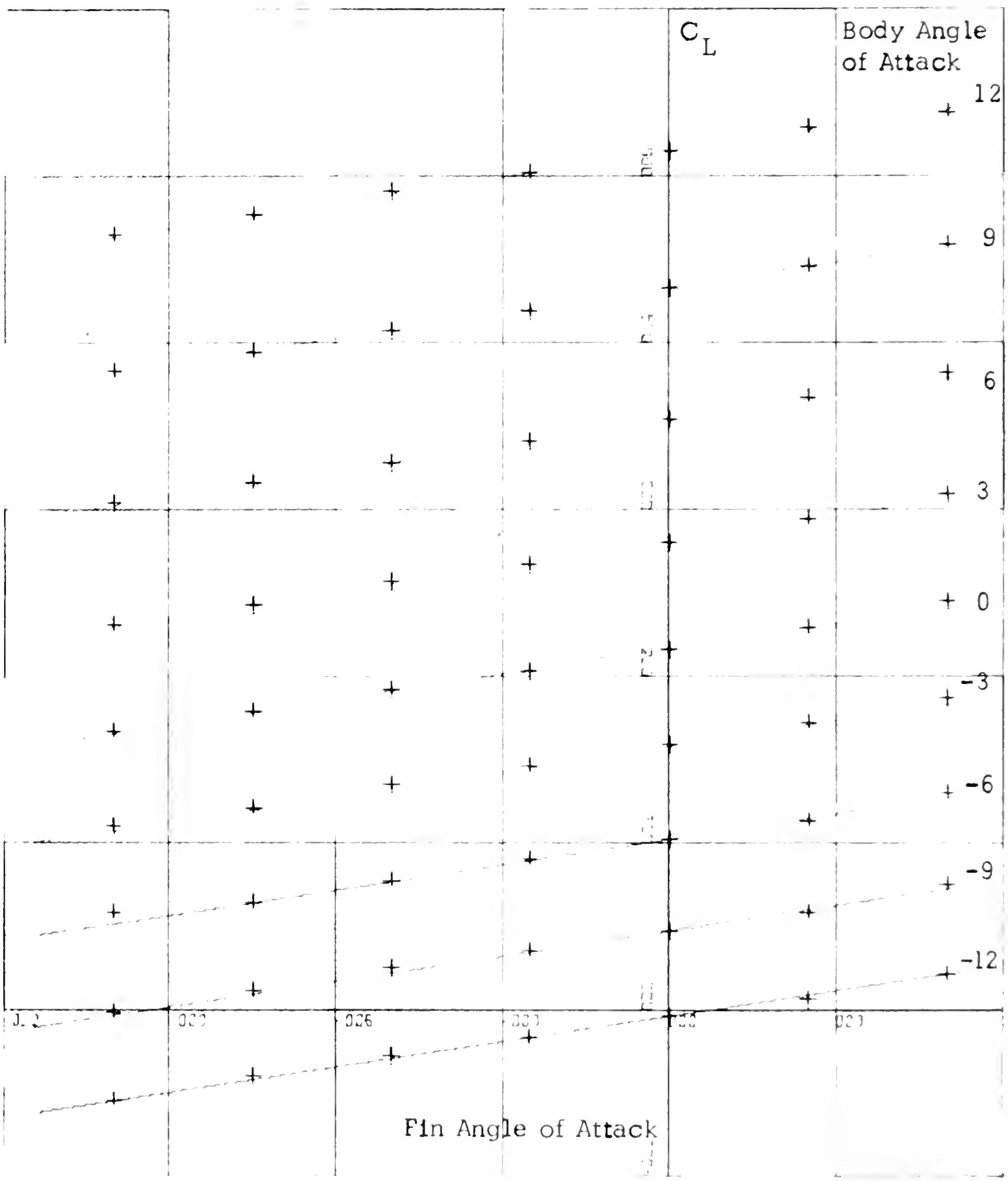
$$\frac{K_A}{s^2 + K_A K_t s + K_A}$$

and the characteristic equation:

$$s^2 + K_A K_t s + K_A = 0 \quad .$$

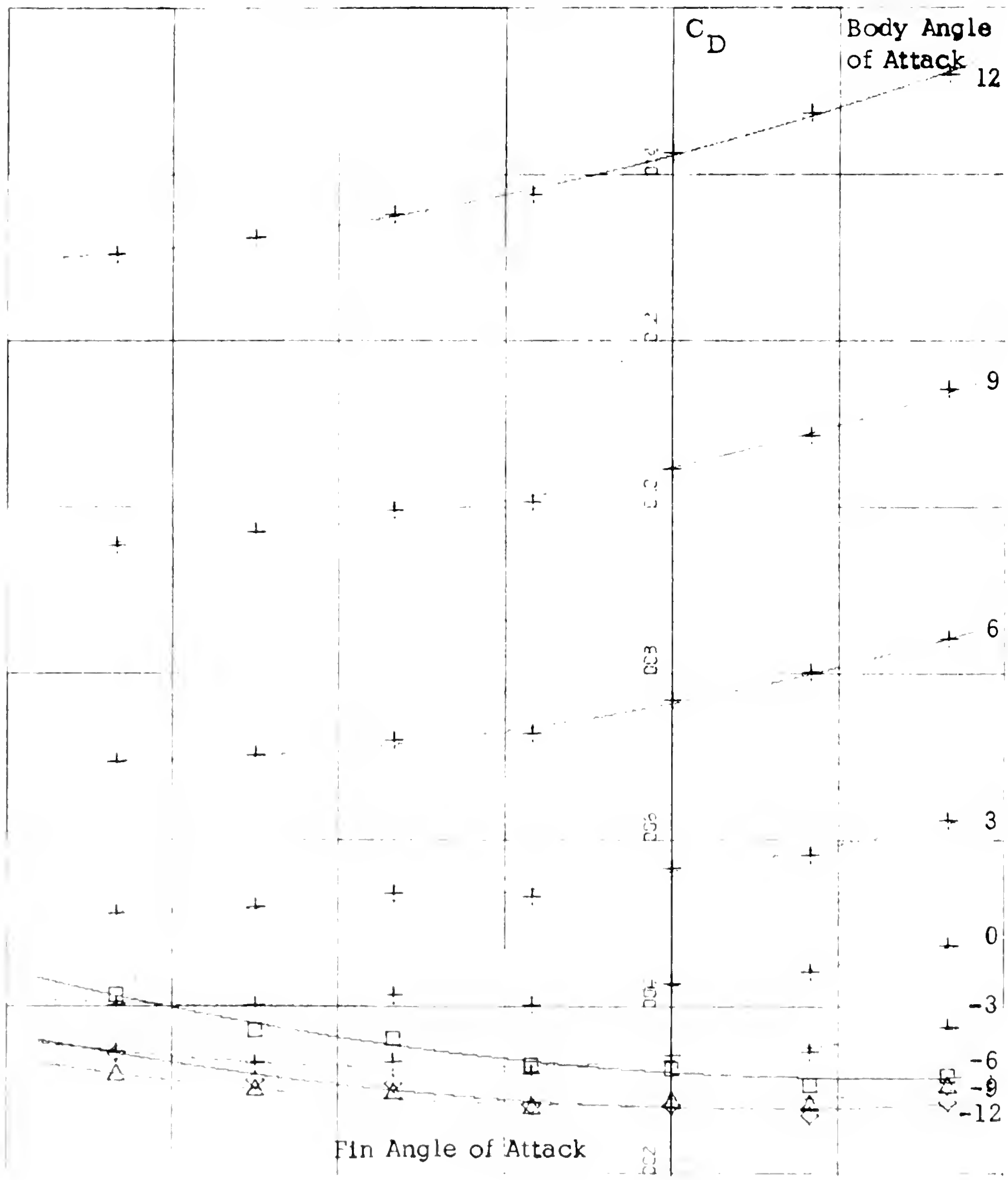
Assuming $K_A = 1000$

$$w_n^2 = K_A = 1000$$



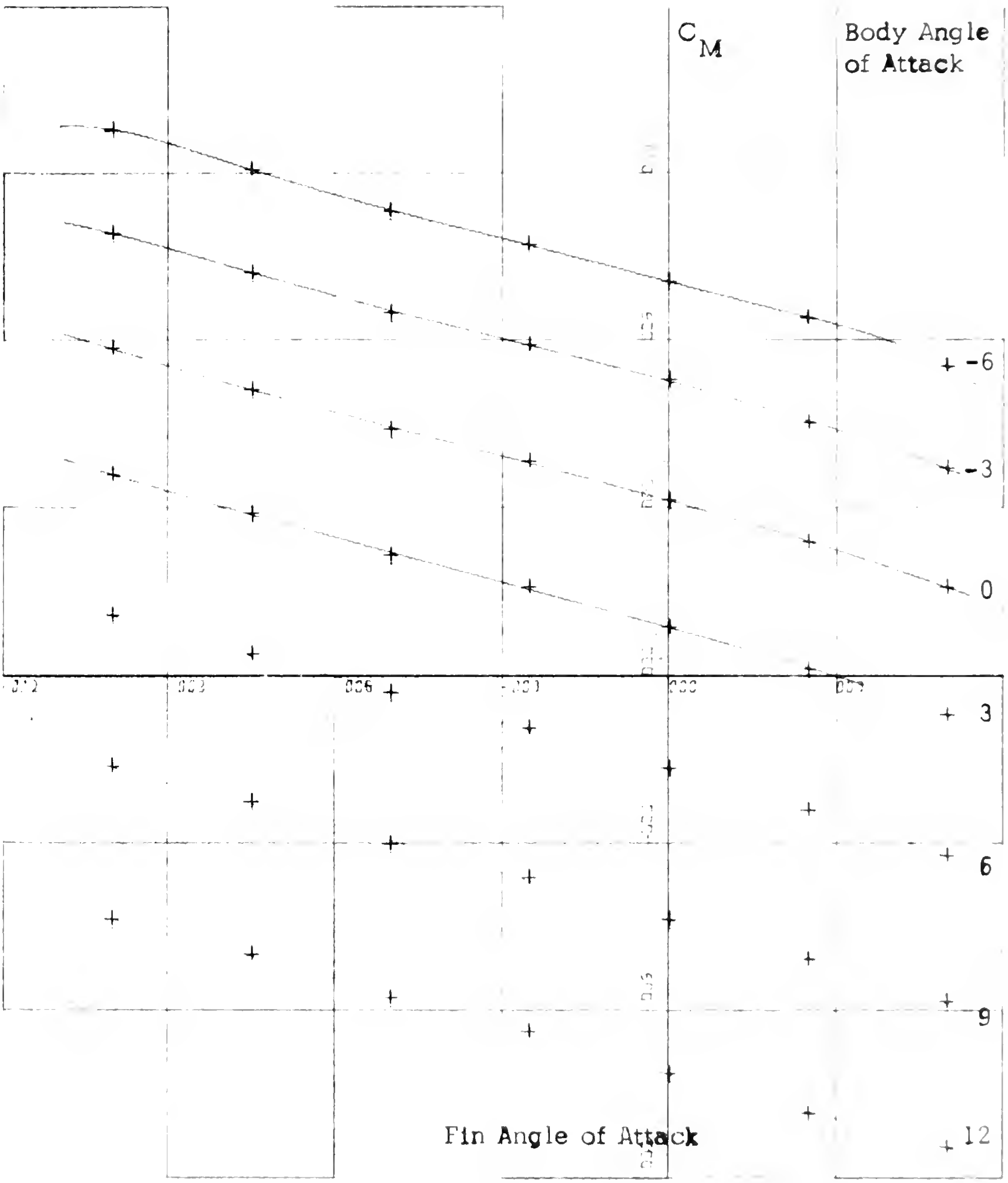
CURVES AND DATA POINTS FOR C_L

Figure 3



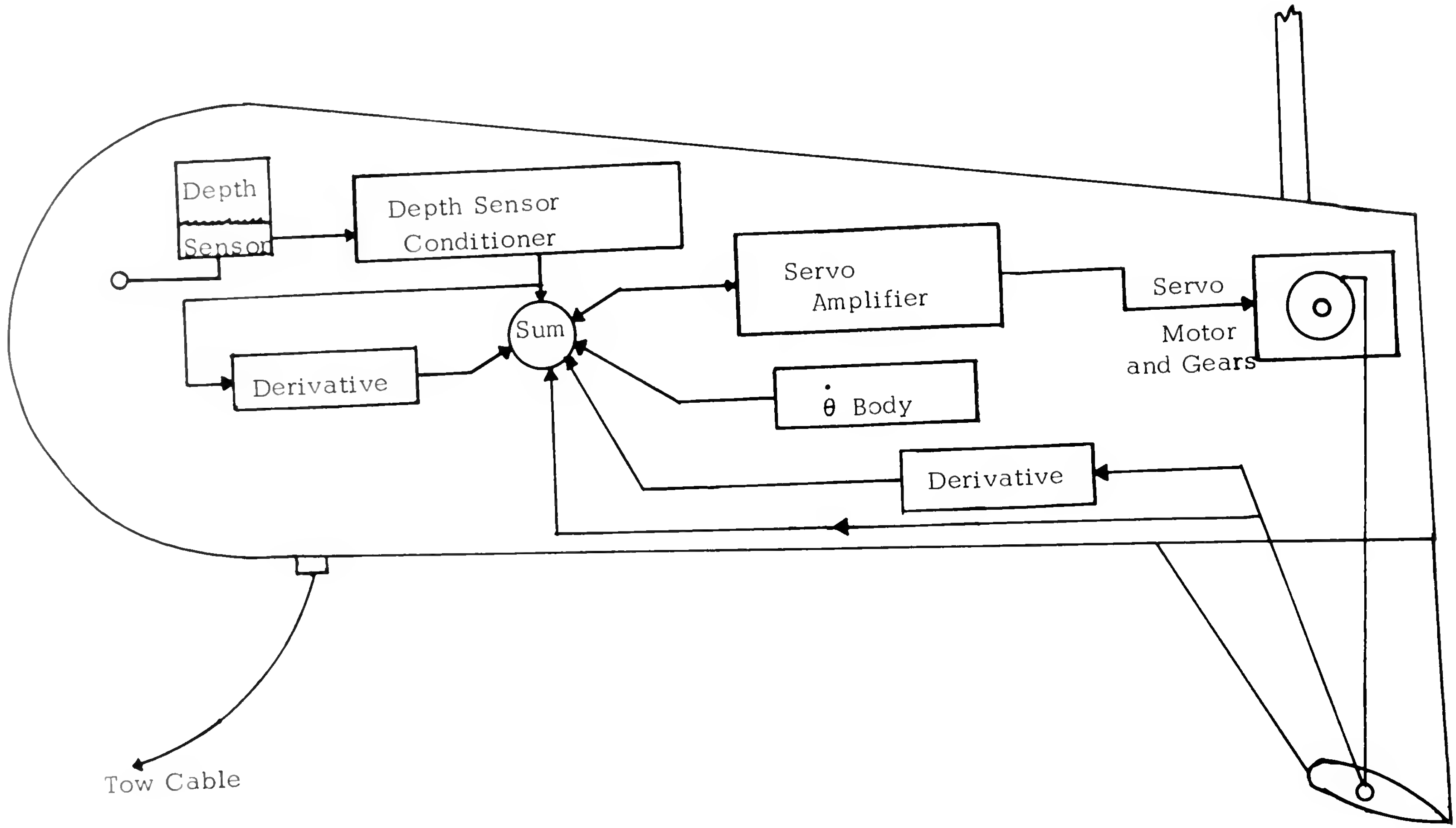
CURVES AND DATA POINTS FOR C_D

Figure 4



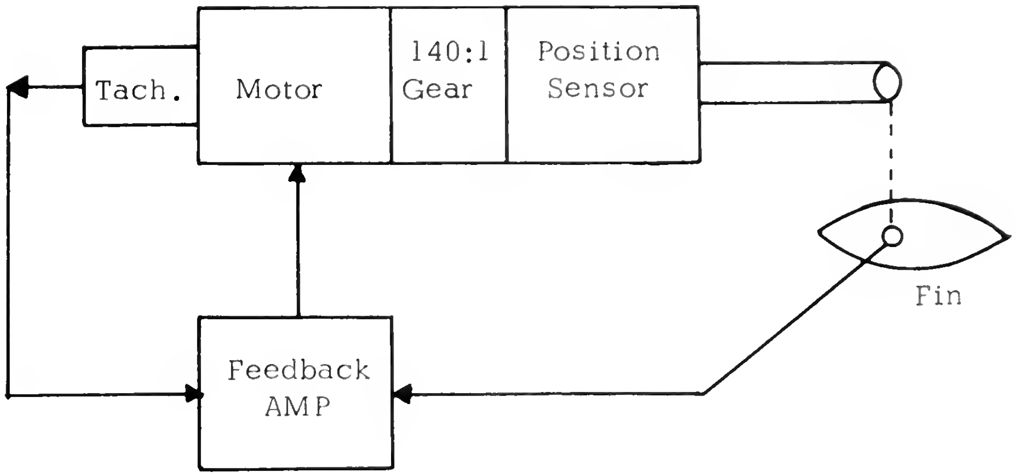
CURVES AND DATA POINTS FOR C_M

Figure 5



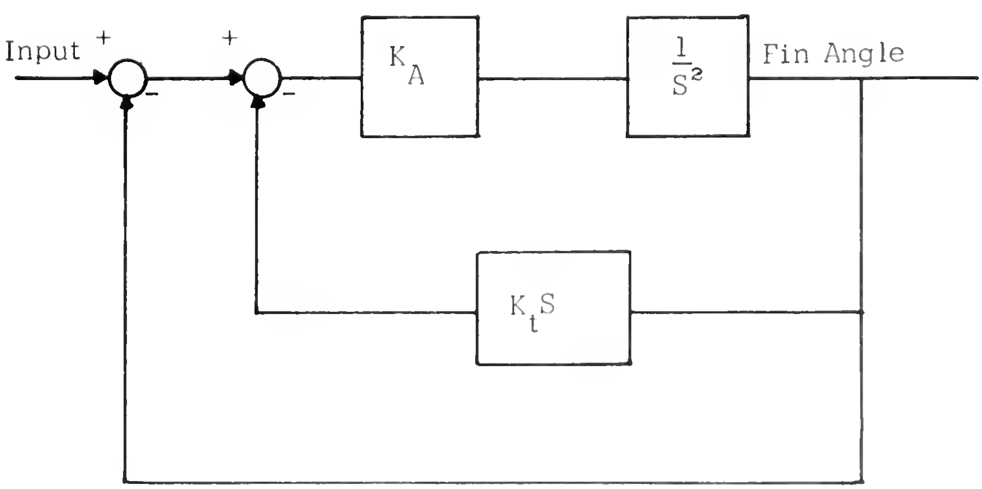
THE FIN SERVO CONTROL SYSTEM

Figure 6



THE FIN SERVO LOOP

Figure 7



THE K/S^2 SERVO SYSTEM

Figure 8

$$2\xi w_n = K_A K_t$$

$$K_t = \frac{2\xi w_n}{1000} \quad .$$

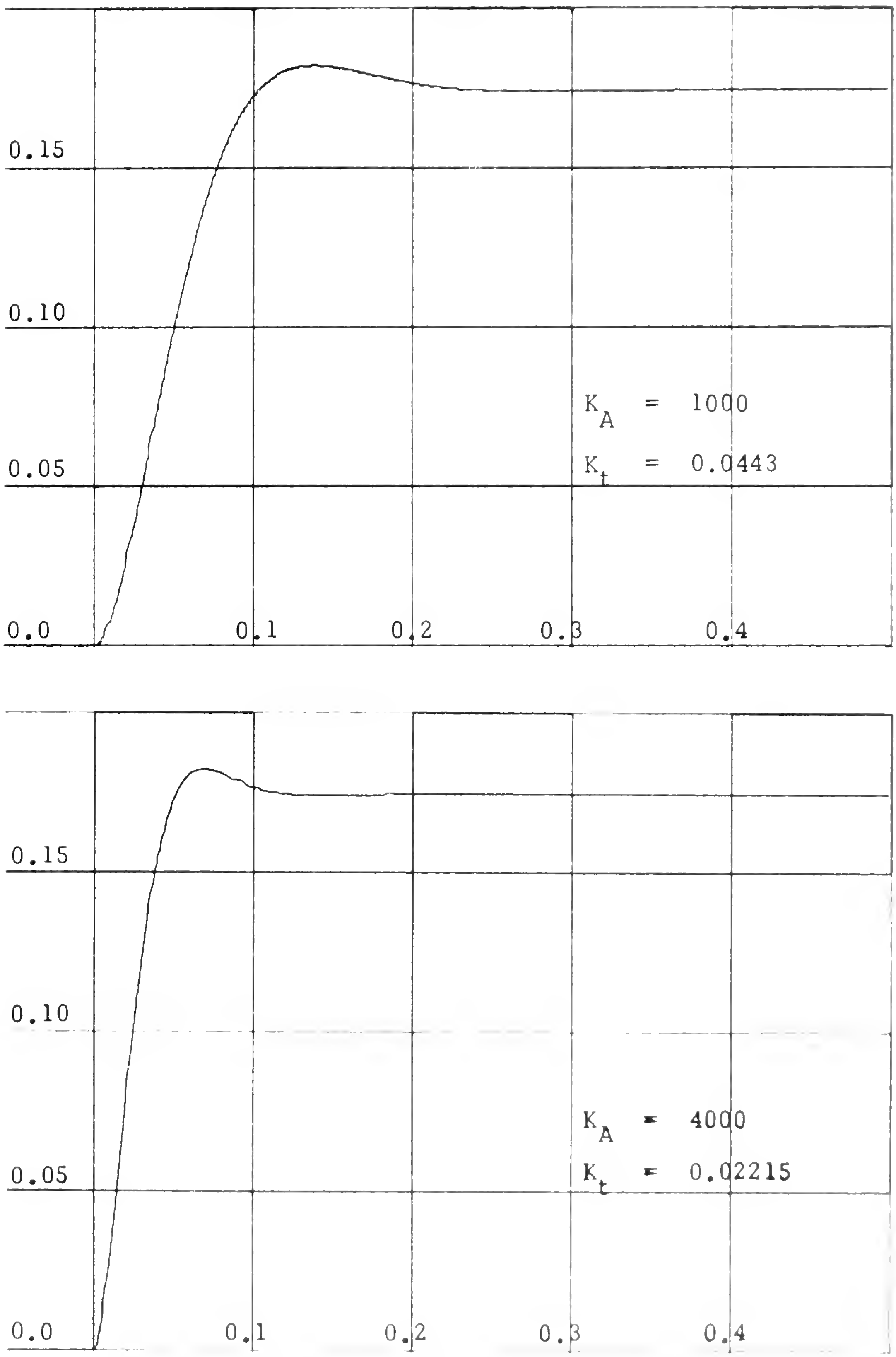
Taking a value of $\xi = 0.7$:

$$K_t = 2(0.7)(31.62)/1000$$

$$= .0443 \quad .$$

With K_A and K_t set at the above values, the rise time of the servo loop is about 0.1 sec. This is not as fast as desired, but by changing the two gains the desired rise time may be obtained. Two different responses of the fin servo loop are shown in Fig. 9; both have an input step of 0.174533 rad. (10 degrees). The first set of gains is used in the model simulation. This is because it is doubtful that the inputs into the servo loop will require that the fin move that fast; if so, then the buoy has probably reached the limits of its surface-following capability.

The inputs into the fin servo loop are shown in Fig. 6. The fin servo loop that was simulated above is composed of the summer, servo amplifier, motor and gears, linkage to the fin, and the unity and tachometer feedback from the fin to the summer. The other three inputs will come from gages and electronic sensors in the body. These inputs are the depth error, the depth rate, and the body pitch rate. In the model simulation, there are three constants (ADE, ADR, and APR) which multiply these three inputs. The values of these three constants are discussed later in the study.



STEP RESPONSES OF TWO K/S^2 SYSTEMS

Figure 9

III. THE BUOY MODEL

In this development of the buoy model, the effect of water currents, the forces generated by the heaving water surface, and the free-surface effect are not taken into account. Currents can be considered neglible compared to the heaving forces the buoy might experience. The free-surface effect is experienced by the buoy because it is a body moving at a depth comparable to its length; a more detailed and complicated model would have to include this force. The force generated by the heaving water surface would not affect the buoy as much as might be expected. This is because the body, as long as its vertical freedom is not restricted, will act like a particle in the water and will move up and down with the surface. Thus, the heaving forces will actually help maintain the buoy near the water surface, which is what is desired. If the buoy were being towed directly above its towpoint, it would have little or no vertical freedom; however, if the buoy is well behind the towpoint, then the cable will provide little restriction to the buoy's vertical freedom. Tests on the actual buoy have shown that with 475 ft. of cable and the two point at a depth of 240 ft., the buoy can heave up to 20 ft. with the waves without the cable decreasing the buoy's vertical freedom of motion by more than 10 percent.[1]

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[1] Soulant, H., Sea Surface Followage by Towed Buoy (Preliminary Study), p. 11, March 1969.

The buoy model will now be developed; the buoy diagram is again produced with some specific dimensions (Fig. 10). Besides the diagrammed dimensions, the buoy has the following additional characteristics:

$S = 14.25 \text{ ft}^2$

$\bar{C} = 6.0 \text{ ft}$

$b = 2.375 \text{ ft}$

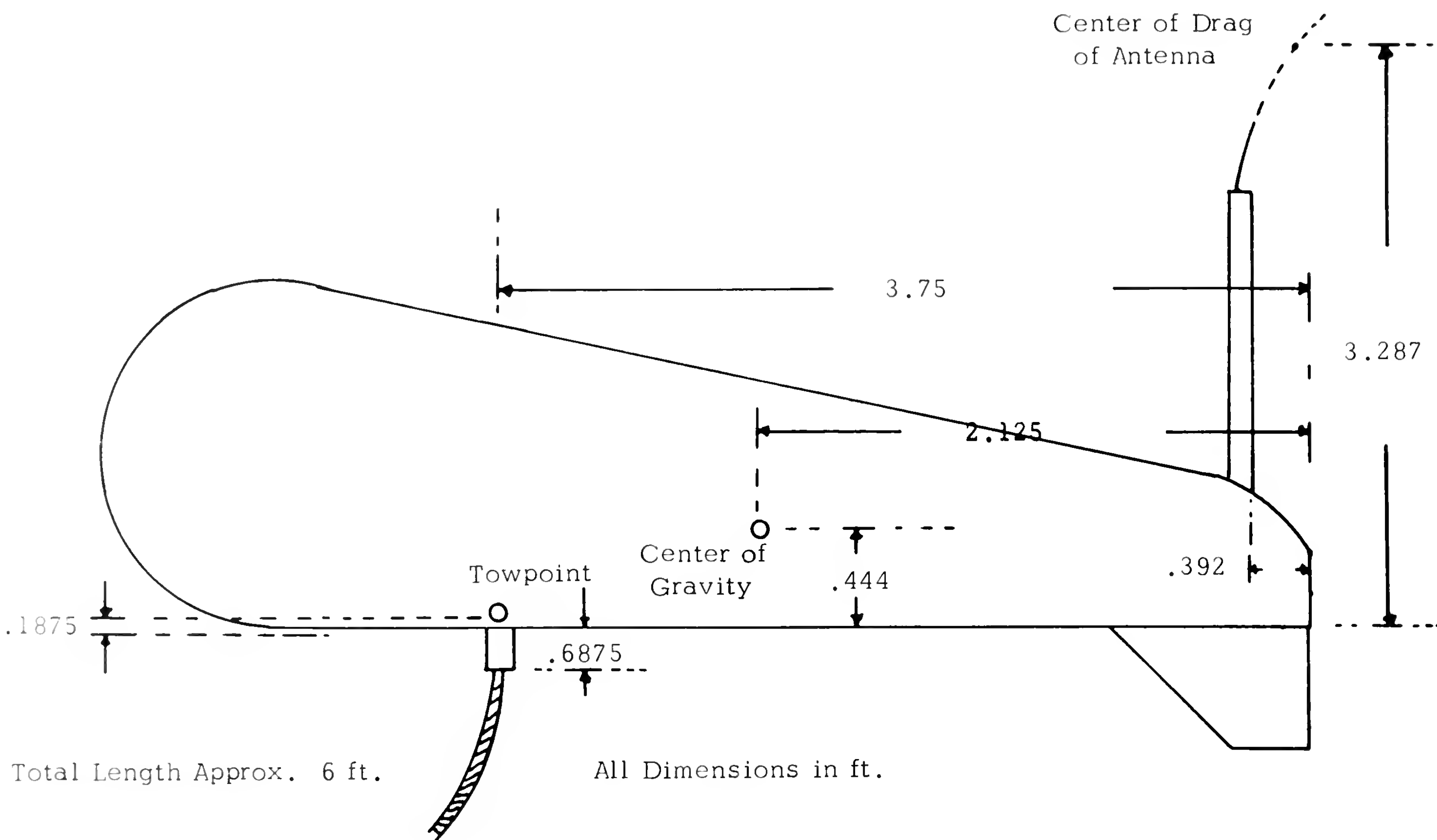
mass = 70 slugs

mass moment of inertia = 975 slug$\cdot$ft^2

volume = 17.5 ft^3

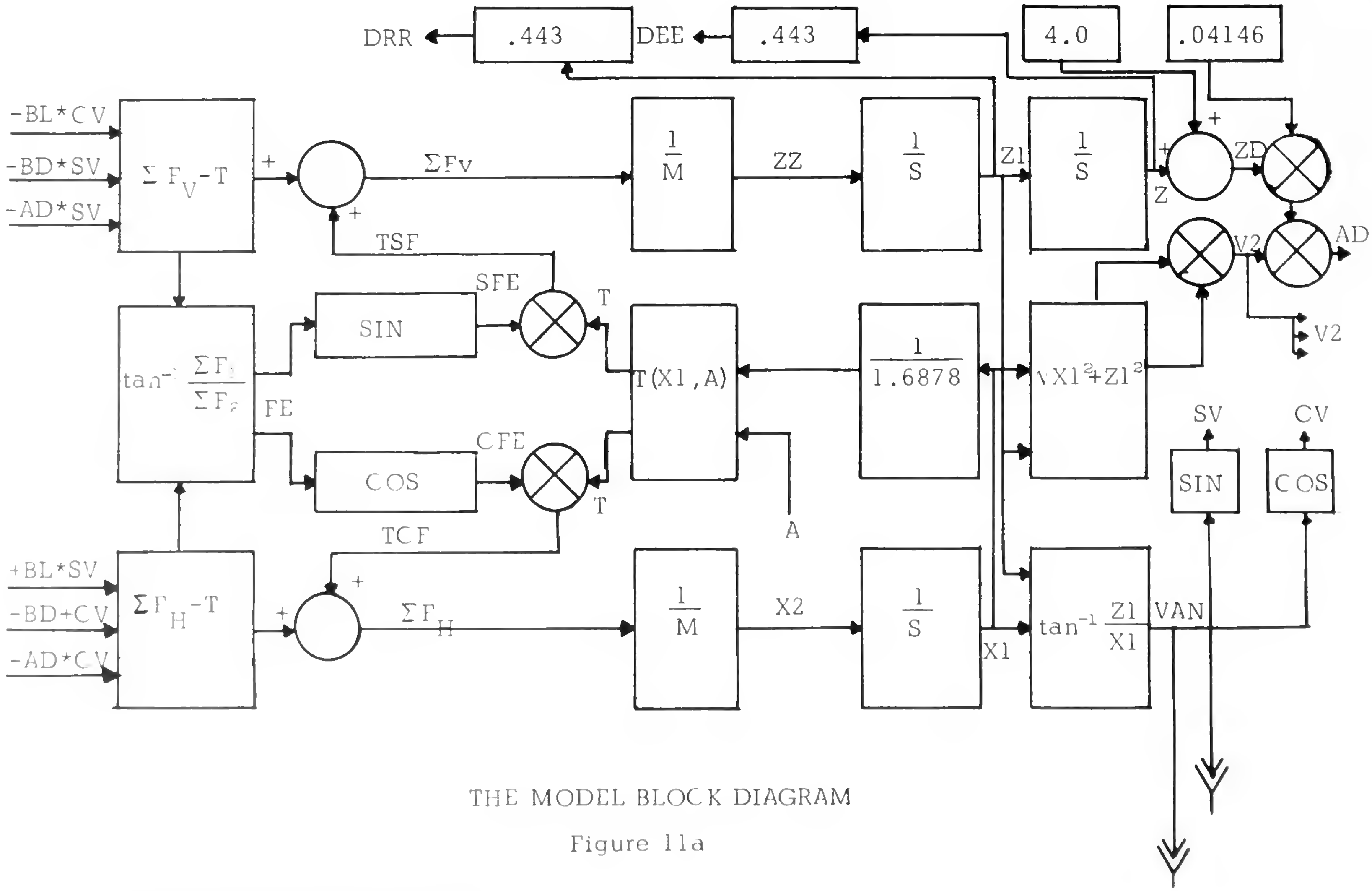
The block diagram of the dynamic model is shown in Fig. 11. As was previously stated, the buoy model is based on the summation of forces in the X and Z planes, and about the Y axis. The summed forces in the X and Z directions are divided by the mass of the buoy, and the moments about the Y axis are summed and divided by the mass moment of inertia. This yields the acceleration in the horizontal and vertical directions, and the angular acceleration about the Y axis. The accelerations are each integrated to give the velocities, and these, when integrated, give the horizontal and vertical position and the angular orientation.

In this first model, the tension force is generated as a function of horizontal velocity and body angle of attack. The tension function was formed in the same manner as the coefficient equations, and was based on predicted values of the tension in the cable. This tension function may also be found in Subroutine Tink of the computer program.



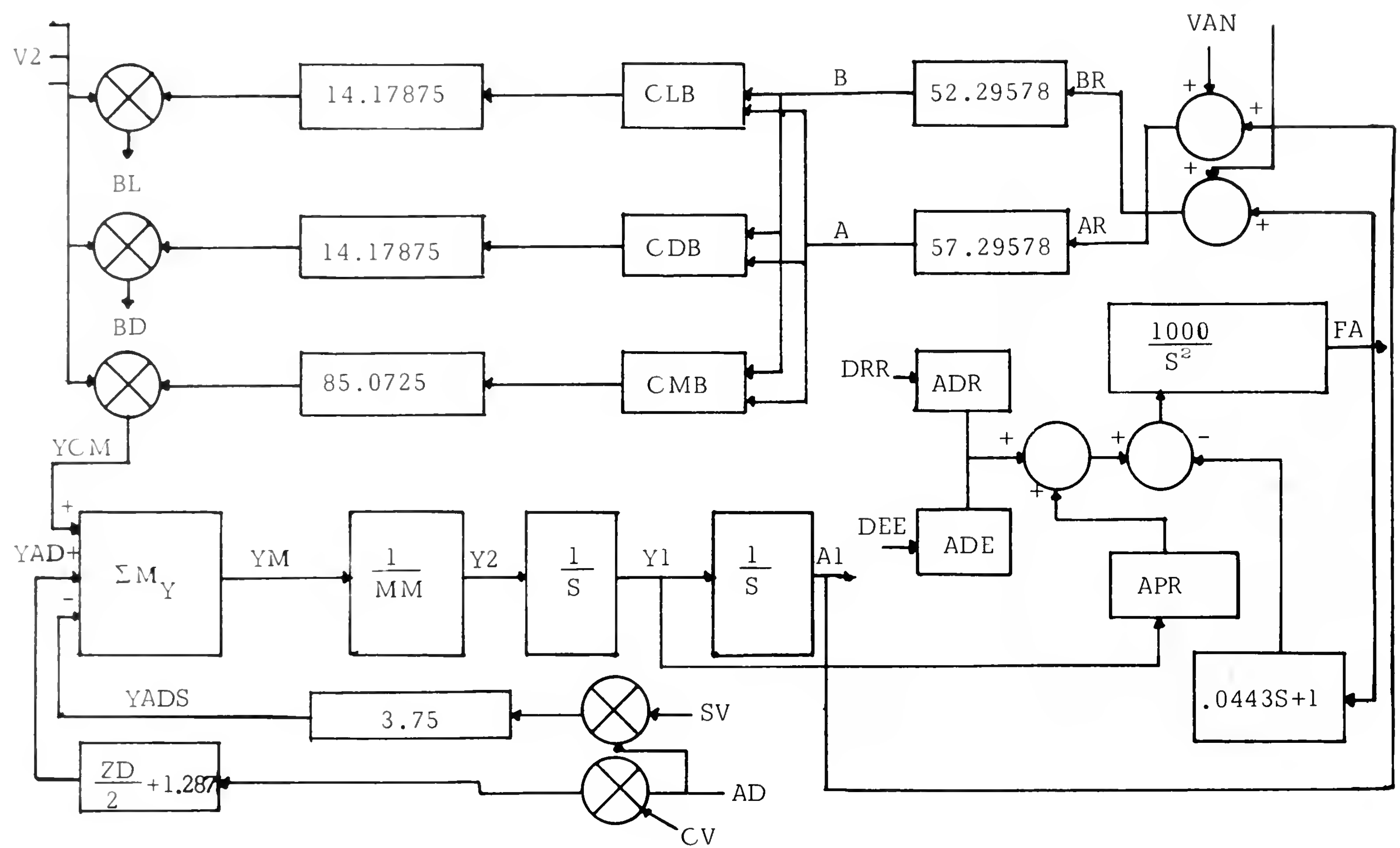
THE BUOY

Figure 10



THE MODEL BLOCK DIAGRAM

Figure 11a



THE MODEL BLOCK DIAGRAM

Figure 11b

The velocities and positions are used to generate the forces and moments that are summed in the basic equations. The forces and moments on a body are relative to the velocity vector of the medium, and not to the local horizontal. Thus, the sum of the squares of the horizontal and vertical velocities yields the relative velocity squared, V2. The arc tangent of the vertical velocity over the horizontal velocity is taken to give the angle of the velocity vector, VAN, with respect to the true horizontal. The angular orientation of the body (angle of attack), A1, is summed with VAN to produce the angle of attack of the body with respect to the velocity vector, AR; AR is in radians. The fin angle of attack, FA, is also summed with VAN to yield the fin angle of attack relative to the velocity vector, BR. These two angles of attack, AR and BR, are multiplied by a conversion factor of 57.29578 deg/rad to yield the angles of attack, A and B, in degrees.

The two angles, A and B, are the arguments which determine the coefficient values of body lift, drag, and moment. The coefficients of lift and drag are both multiplied times V2 and a constant to produce the body lift, BL, and the body drag, BD. The constant is $\frac{1}{2}\rho S$, and is equal to 14.17875.

The moment coefficient is multiplied times V2 and a constant, $\frac{1}{2}\rho S\bar{C}$; this constant is equal to 84.0725. This body moment, YCM, is summed with the moments produced by the antenna, to yield the total moment, YM, acting on the body about the towpoint.

There is no definite information on the antenna, but it is assumed that it will only contribute a drag force; this drag force will contribute to the total drag and moment on the body. The antenna coefficient of drag is assumed to remain constant at a value of 0.50. Thus, the antenna drag is dependent on a constant times the frontal surface area times the relative velocity squared. Assuming a frontal width of 2 inches, the drag force of the antenna will then vary with the length of antenna under water, and V2. The length of antenna will be approximated as the depth of the body below the surface. The constant in this case is $\frac{1}{2}(1/6)\rho C_{DA}$, which equals 0.04146. The antenna drag is multiplied by the sine and cosine of VAN, SV, and CV, to give two moment components. These times their respective moment arms yield YADS and YADC.

The body lift and drag, and antenna drag are multiplied by SV and CV, and the resulting six values are summed into the horizontal and vertical forces. In this first realization of the model, these horizontal and vertical forces are used before being summed with the horizontal and vertical components of the tension force, to determine the angle FE with which the tension force acts on the body. This means that the tension force acts exactly opposite to the vector of body forces; this simplifying assumption is used in the initial stages of the model development.

The depth error and depth rate inputs into the fin servo loop are taken from the vertical position and velocity, Z and Z1, respectively. These are multiplied by a constant, 0.443, which converts ft and ft/sec into psi/ft and psi sec/ft. These are multiplied by gain constants ADE and

ADR before being summed with the body pitch rate, Y1, times its gain constant, APR. These three inputs are fed into the fin servo loop. The orientation of the fin causes a torque on the body which either increases or decreases the body angle of attack; this enables the body to rise or descend.

This is the initial dynamic model; it changes as more insight and knowledge are gained on the model. The first study with the model deals with the three coefficients ADE, ADR, and APR.

IV. STUDY OF THE BUOY MODEL

Once the model was digitally programmed and the initial programming mistakes ironed out, the first step was to study possible values of the fin servo input coefficients. The first computer program realization is shown in the program titled "INITIAL BUOY MODEL". The buoy is initiated at a depth, ZI, of 4.0 ft, a speed, X1I of 20.2536 ft/sec (12 knots), body angle of attack, A1I, of 0.10472 rad (6 degrees), and a fin angle of attack, FAI, of 0.02 rad (1.15 degrees). The tension force and the coefficients of lift, drag, and moment are called out of Subroutine Tink, which is in double-precision format.

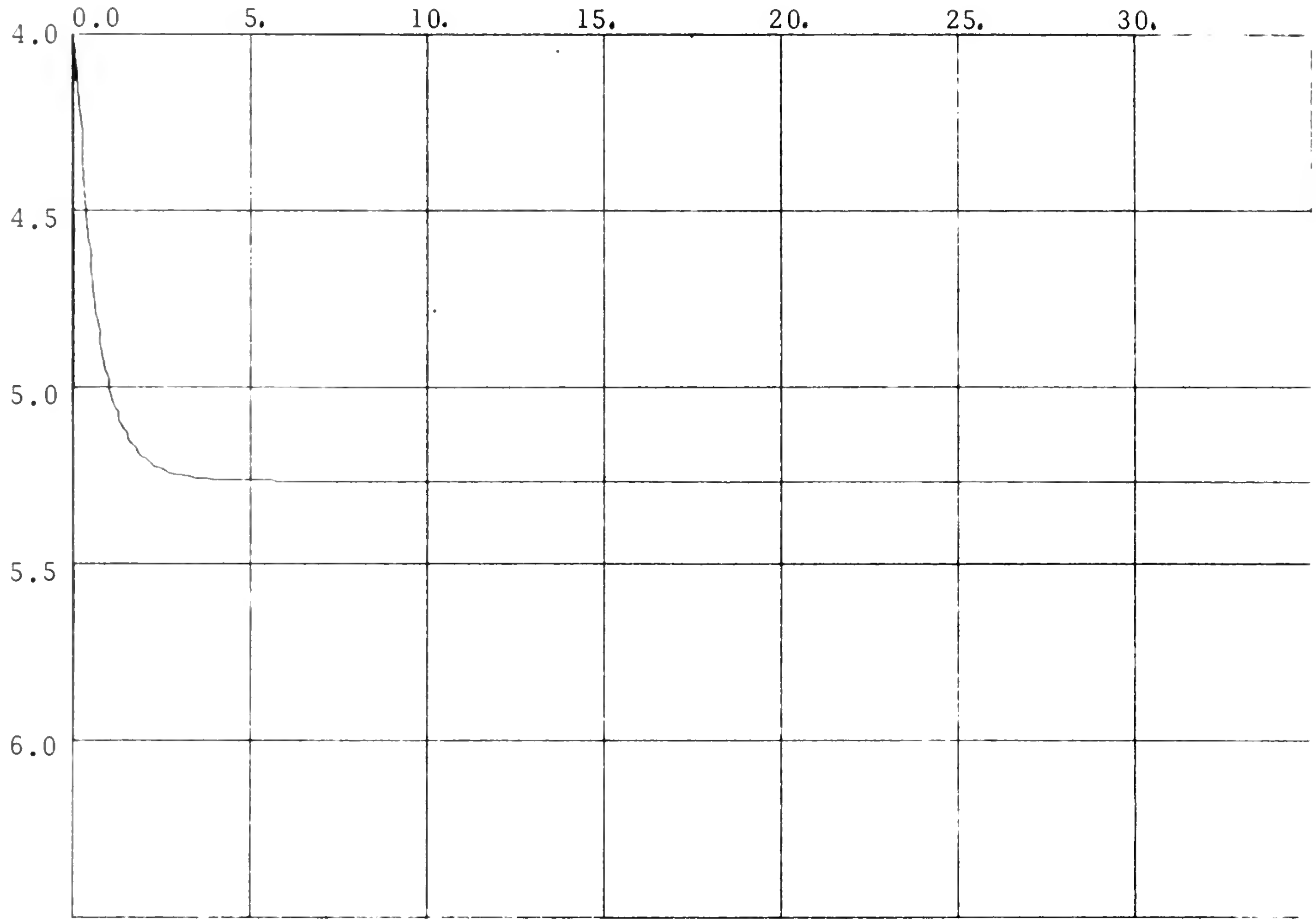
A. SERVO INPUT COEFFICIENTS STUDY

The first attempt in this study was to see if the buoy would stabilize with only one input into the fin servo loop; either the depth error, the depth rate, or the body pitch rate. This proved fruitless, so different combinations of ADE and ADR were tried, and it was found that these two, with APR set equal to zero, would stabilize the system. A full range of values of ADE and ADR were tried to see which combinations would bring the buoy to a steady-state position. A mapping of these values is shown in Table IV; the value given for each combination is the depth at which the buoy steadied out. The responses for several of these combinations are shown in Figs. 12-16. It appears from the depth values that the tension force is too strong, since in every case the buoy steadies out deeper than the desired depth of 4.0 ft. This became obvious when the

ADR \ ADE	0.3	0.5	0.7	0.9	1.1	1.2	1.3	1.4	1.6	1.8	2.0	2.5	3.0
0.05						4.599	4.547	4.511					
0.06					4.656	4.587	4.539	4.502					
0.07					4.637	4.579	4.533	4.496					
0.08					4.627	4.572	4.527	4.492	4.454	4.388			
0.09					4.691	4.567	4.524	4.488	4.433	4.385			
0.10				4.778	4.614	4.563	4.520	4.485	4.427	4.383			
0.20		5.292	4.922	4.719	4.591	4.543	4.502	4.467	4.411	4.367	4.332		
0.30	6.101	5.270	4.911	4.712	4.584	4.536	4.496	4.462	4.405	4.362	4.327	4.265	4.255
0.40		5.268	4.909	4.710	4.583	4.536	4.495	4.461	4.404	4.360	4.325	4.263	4.227
0.50			4.909	4.711	4.593								
0.60				4.712	4.586								
0.70					4.537	4.496	4.469						

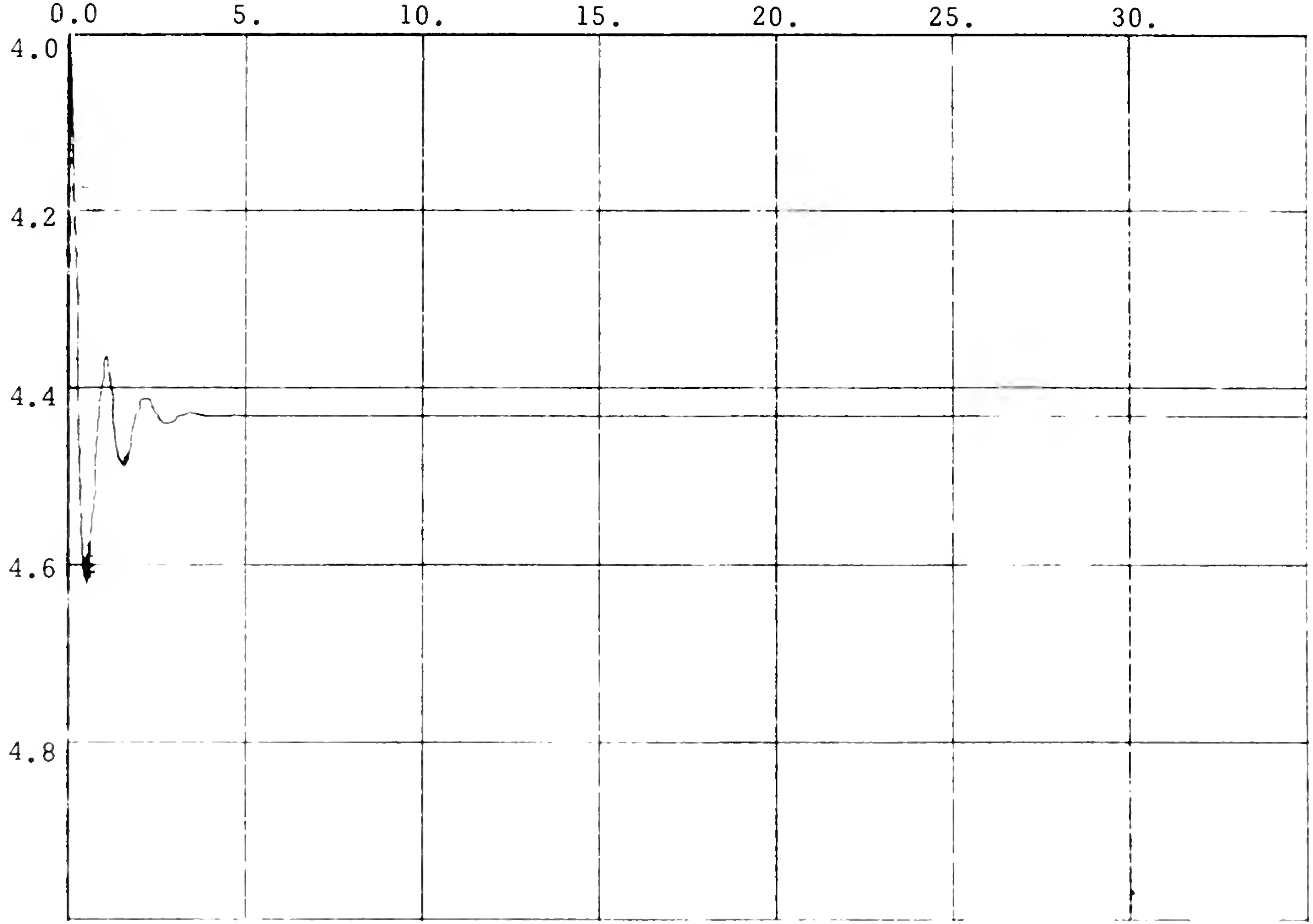
MAPPING OF STABLE COMBINATIONS OF ADE AND ADR

Table IV



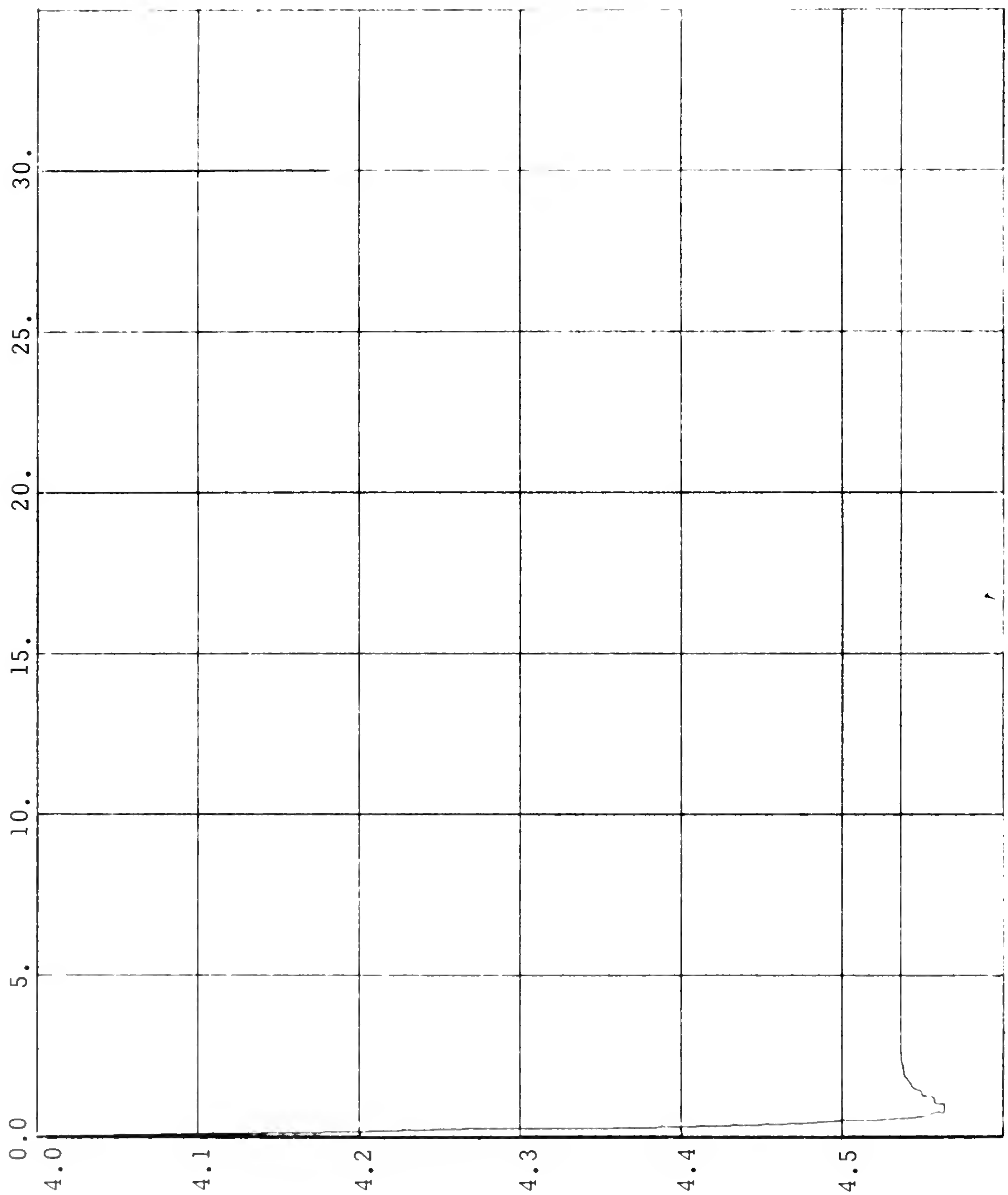
RESPONSE OF ADE=0.50 AND ADR=0.3

Figure 12



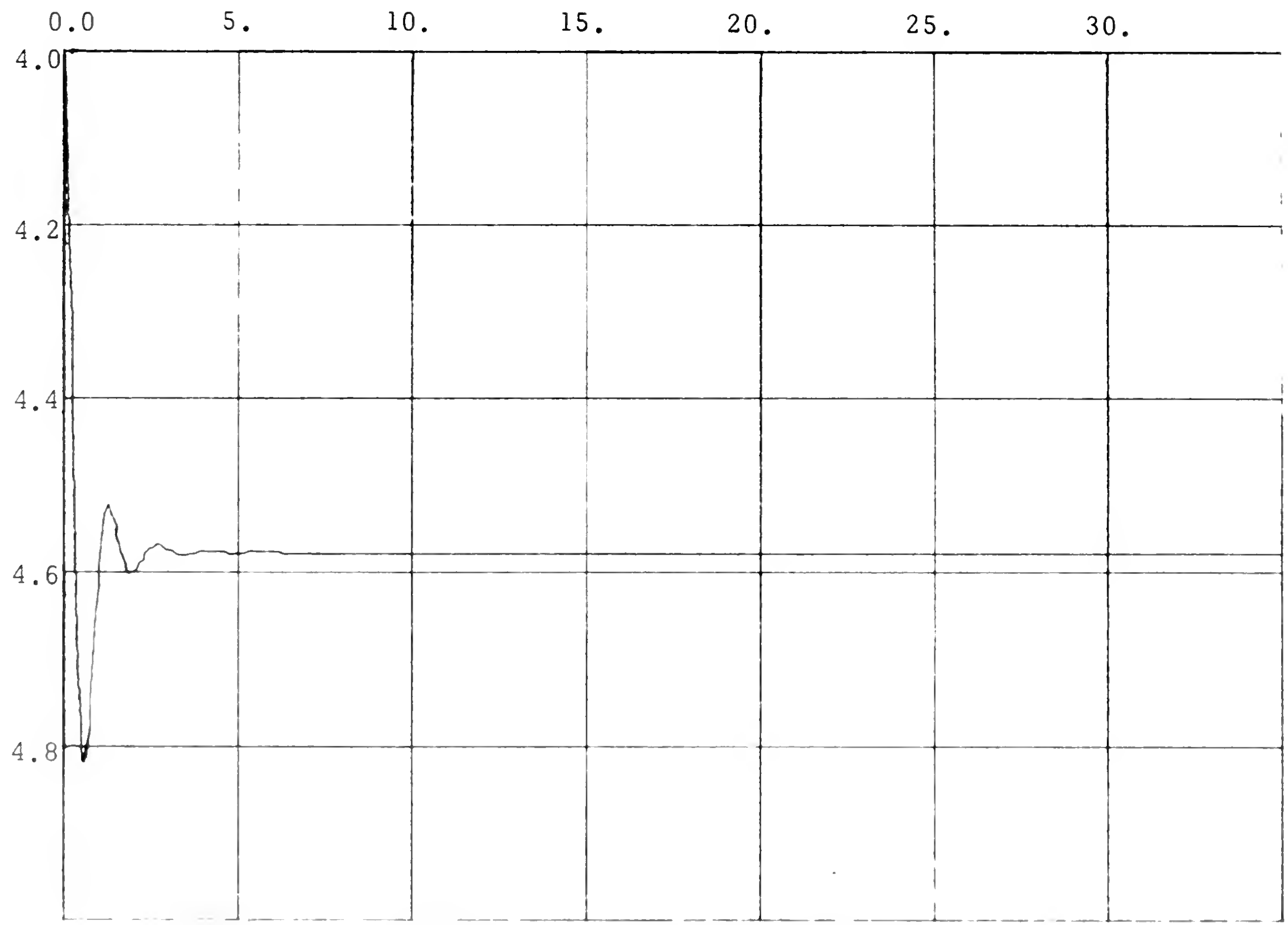
RESPONSE OF ADE=1.6 AND ADR=0.09

Figure 13



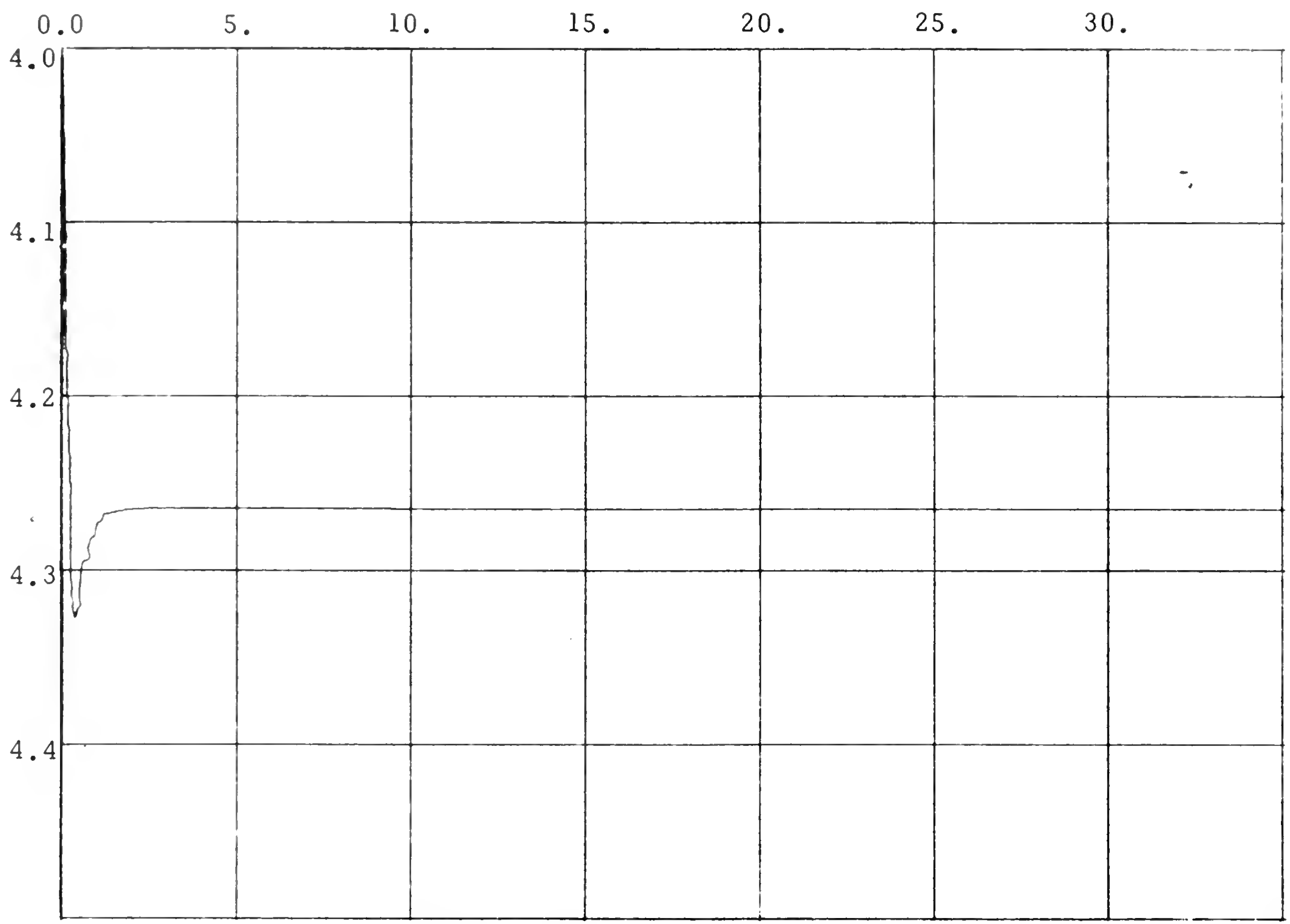
RESPONSE OF ADE=1.2 AND ADR=0.3

Figure 14



RESPONSE OF ADE=1.2 AND ADR=0.07

Figure 15



RESPONSE OF ADE=2.5 AND ADR=0.3

Figure 16

steady-state body angle of attack and fin angle of attack were checked; both angles were large and showed that the model was working properly, but that the buoy was fighting against an unrealistic tension force formulation.

It was decided from the mapping of the two coefficients, ADE and ADR, that three sets would be used in further studies of the buoy model. The first pair was chosen on the basis that it was on the intersection of the values of ADE and ADR which had the widest range of stable values, and that this set was towards the center of the mapping. This was ADE-1.2 and ADR=0.30. It was then decided that the other two pairs would vary in one coefficient away from this point. These other two pairs are ADE=1.2 and ADR=0.07, and ADE=2.5 and ADR=0.30. The responses with these three pairs of coefficients are shown in Figs. 14-16.

The coefficient APR remained to be studied. When this coefficient was given a value larger than zero there was no appreciable change in the response of the buoy. To check the range of possible values of APR, the tension force, T, was changed so that it was equal to the square root of the sum of the squares of F1 and F2. Thus, the buoy would stay at four feet below the surface and the effect of APR could be seen in the response of the buoy's angle of attack. Both negative and positive values of APR were tried, and negative values immediately drove the buoy model unstable. Positive values of APR, up to 7.0, were tried with the three pairs of depth error and depth rate coefficients. At this upper value of

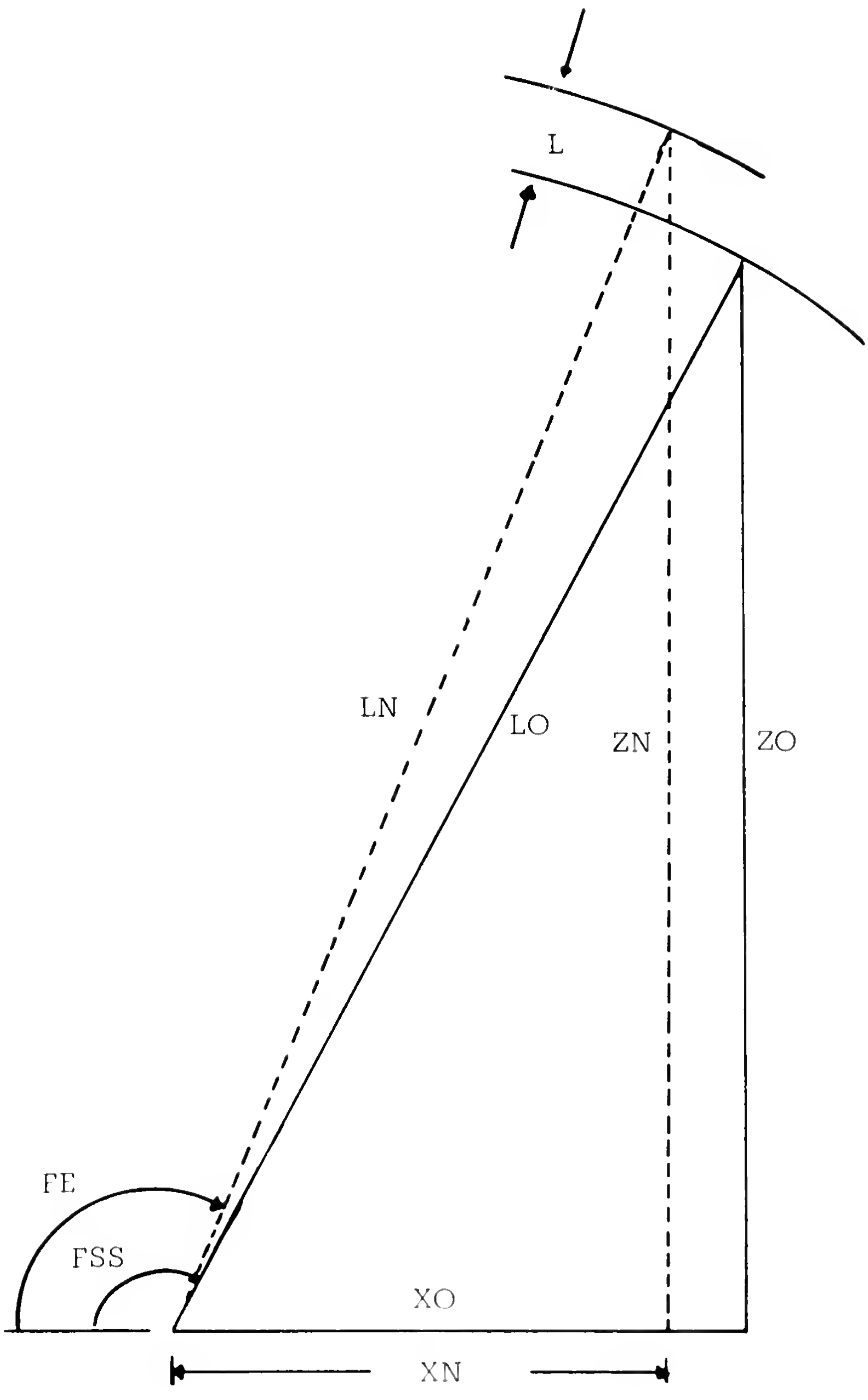
7.0, the body started to oscillate away from stability about the Y axis; it was apparent that this high value of gain was approaching the stability limit of the buoy model. Based on this, a median value of 3.0 was chosen for APR. The three final sets of coefficients are as follows:

ADE	ADR	APR
1.2	0.07	3.0
1.2	0.30	3.0
2.5	0.30	3.0

B. TENSION FORCE FORMULATION

The next step in this model study was to formulate a new, and hopefully more realistic, tension force, rather than using the predicted tension data supplied with the buoy. It was decided that this tension function would be a constant plus or minus the distance away from the equilibrium point times what will be called a spring constant, SC. The basis for this formulation is shown in Fig. 17. First it is assumed that the last portion of the cable connected to the buoy, LO, is straight, and that of the total length of the cable this section is the most dynamic. By computing the increase or decrease in ZO and XO, a new cable length, LN, may be calculated. If LN is larger than LO then the tension increases; if it is smaller then the tension decreases. The difference in length (LN-LO) is L and this is multiplied times the spring constant, SC, which is studied for several different values later on. The new tension force now takes the form:

$$T = 1940 + L \cdot SC \quad .$$



THE NEW TENSION FORCE FORMULATION

Figure 17

The value 1940 is the steady-state tension value that the model settled to in the earlier parameter studies; the angle FSS is the steady-state value of FE. This new tension force was implemented into the buoy model. The buoy will now be initiated at a depth of zero feet; this is possible because no heaving force or surface effect is taken into account in the model; other minor changes are made to take this new initial condition into account. The new computerized model program is titled "STUDY OF T = 1940 + L*SC". The body angle is now being initiated at 4.8 degrees and the initial fin angle is 0.0 degrees; these two values are also the steady-state values for these angles found earlier. The next step in the formulation of this tension force was to find an appropriate value for the spring constant. The program was run with the three sets of coefficients and the new initial conditions, while SC was varied from 2000. to 0.0 in steps of 250. The two sets of coefficients with ADE=1.2 were stable for all values down to SC=250, while the set with ADE=2.5 was stable down to SC=1750. This third set (ADE=2.5, ADR=0.30, and APR=3.0) was dropped from further investigation since this was believed to be an unrealistic value for the spring constant.

C. STUDY OF INITAL CONDITION PERTURBATIONS

This final form of the model with the new tension force formulation was next examined to see how stable the model was under various initial condition perturbations. It was decided to perturb the initial vertical velocity, the body angle of attack, and the fin angle of attack. Three different spring constants were used with the two remaining sets of coefficients; these were SC=500, SC=1000, and SC=1500. It was also decided that two different values for the approximation of the dynamic length of the cable would be used; these were ZO=100 and ZO=50. (ZO simply because it was used to initiate the model rather than LO). The final results were tabulated, and are shown in Table V; the initial angles of attack (A1I and FAI) are in degrees, and the initial vertical velocity (Z1I) is in ft/sec.

The responses of the buoy model to some of these initial condition perturbations are shown in Appendix A. The twenty-four plots include all the maximum perturbations listed for SC=500 and SC=1500. These plots do not really tell much about the buoy model. They do show that the buoy will settle to steady state fairly rapidly, and that the buoy will not deviate very far from its desired depth. Plots with the same pair of coefficients (ADE and ADR), with the same dynamic length of cable, and with the same type of inital condition perturbation, but with different values of spring constant, seem to be very similar. The responses with the larger spring constant (SC=1500) seem to settle more quickly, even though they are oscillating through a wider range of depths. All the responses to the fin angle perturbations are remarkably similar. This might be because the fin is quickly driven back to the desired stability position; and since all the fin angle perturbations are the same pair of values, this similarity in responses is not surprising. The perturbing of the initial conditions for the new tension force realization was the point at which this study ended.

	ZO=50.		ZO=100.	
	ADE=1.2 ADR=0.07 APR=3.	ADE=1.2 ADR=0.3 APR=3.	ADE=1.2 ADR=0.07 APR=3.	ADE=1.2 ADR=3.0 APR=3.
SC=500				
AlI	-2.1→+0.9	-1.8→+0.9	-1.7→+1.	-1.5→+0.9
AlI	+2.5→+8.0	+3.0→+7.5	+2.0→+7.	+2.5→+6.5
FAI	-17.5→+15.0	-17.5→+15.0	-17.5→+15.0	-17.5→+15.0
SC=1000				
ZlI	-4.8→+2.8	-2.9→+2.6	-4.3→+2.5	-2.8→+2.2
AlI	-0.5→+9.5	0.0→+9.0	-0.5→+8.0	0.0→+8.0
FAI	-17.5→15.0	-17.5→+15.0	-17.5→+15.0	-17.5→+15.0
SC=1500				
ZlI	-4.8→+3.1	-2.7→+3.3	-4.8→+3.0	-2.7→+2.9
AlI	-2.0→+10.0	-1.0→+9.5	-1.5→+9.0	-1.0→+8.5
FAI	-17.5→+15.0	-17.5→+15.0	-17.5→+15.0	-17.5→+15.0

STABILITY RANGE OF INITIAL CONDITIONS

Table V

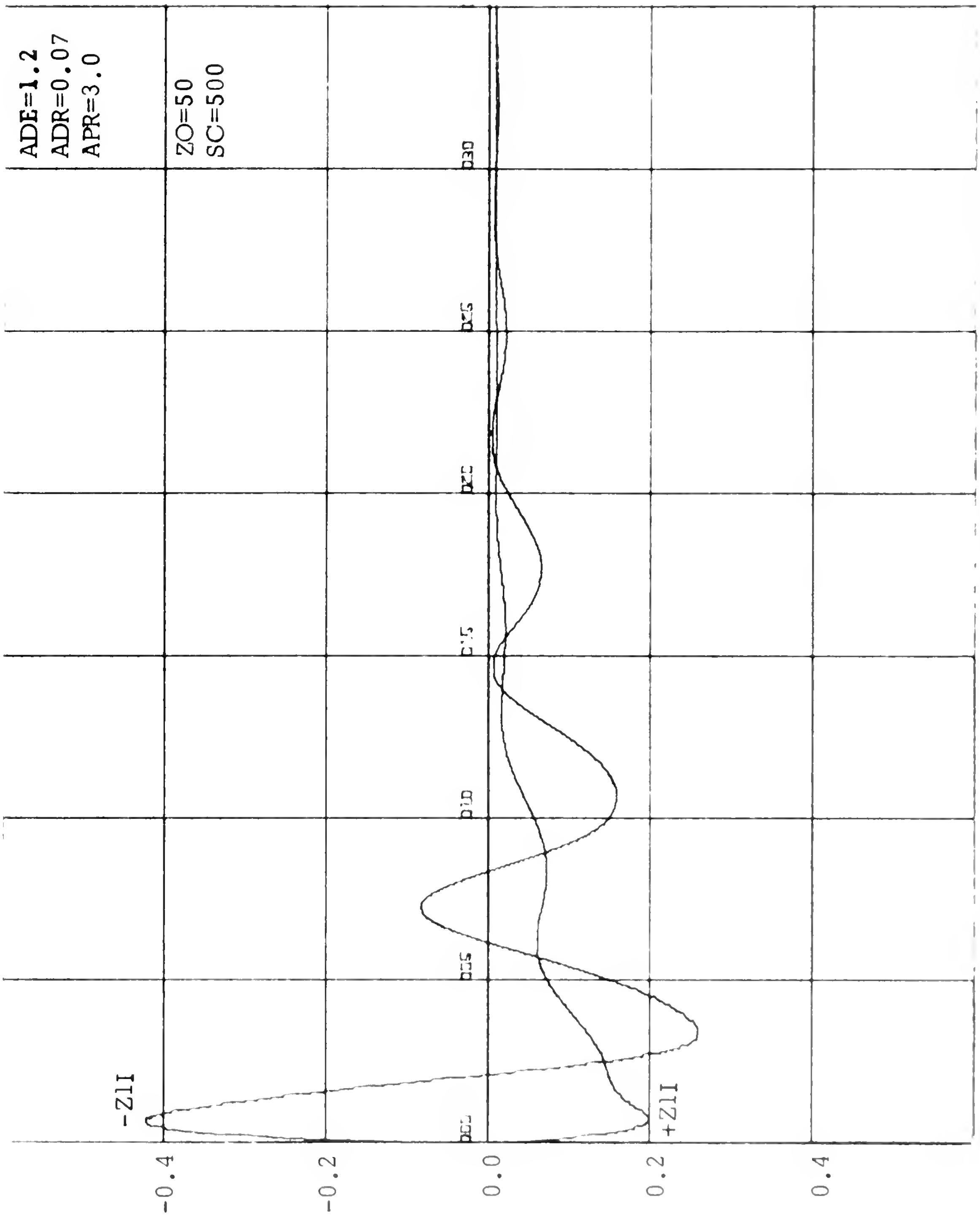
V. CONCLUSIONS AND RECOMMENDATIONS

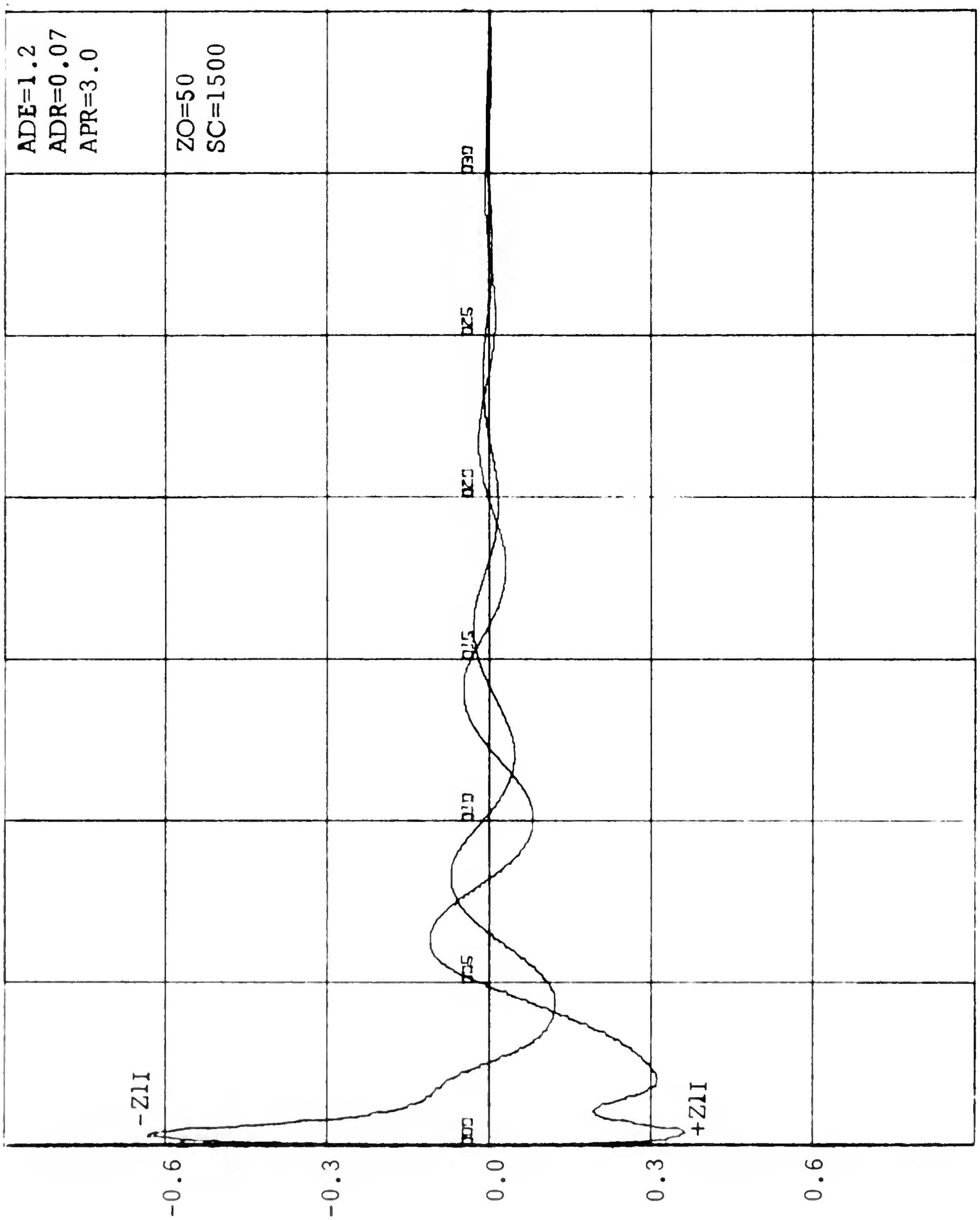
The study presented here is only a beginning to this problem. There are many facets and additions to this problem that remain to be studied. First of all, the dynamic characteristics and wave-following capabilities of this model were never investigated. Other additions could be made to the model to take into account the heaving forces generated by the waves and the free-surface effect which will definitely affect the motion and capabilities of the buoy. There remains much to be studied in the area of the dynamic cable and the coupling of the tension force to the buoy model. Other tension force realizations might be tried with several spring constants in a single cable length; or the cable might be approximated by a series of straight lengths, each with its own spring constant. The possibilities of adding a damping coefficient into the tension force realization need to be investigated; the possibilities are almost endles. Other methods of simulating the buoy need to be tried, especially that of using a hybrid computer (an analog computer integrated with a digital computer); hybrid computers are more suited to this type of real-time simulation work. A dynamic model that allowed for all the possible dynamic forces and all the possible variations in buoys and cables, would be invaluable in the design and engineering of towed buoys.

APPENDIX A

RESPONSES OF THE BUOY TO INITIAL CONDITION PERTURBATIONS

The following 24 plots are the responses of the buoy to the maximum initial condition perturbations listed in Table V for SC=500 and SC=1500. The plots are of depth versus time. The x scale for all the plots is 5 sec/inch; and the y scales are in ft/inch.


ADE=1.2
ADR=0.07
APR=3.0
ZO=50
SC=500
-Z1I
+Z1I
-0.4
-0.2
0.0
0.2
0.4


ADE=1.2
ADR=0.07
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ZO=50
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-Z1I
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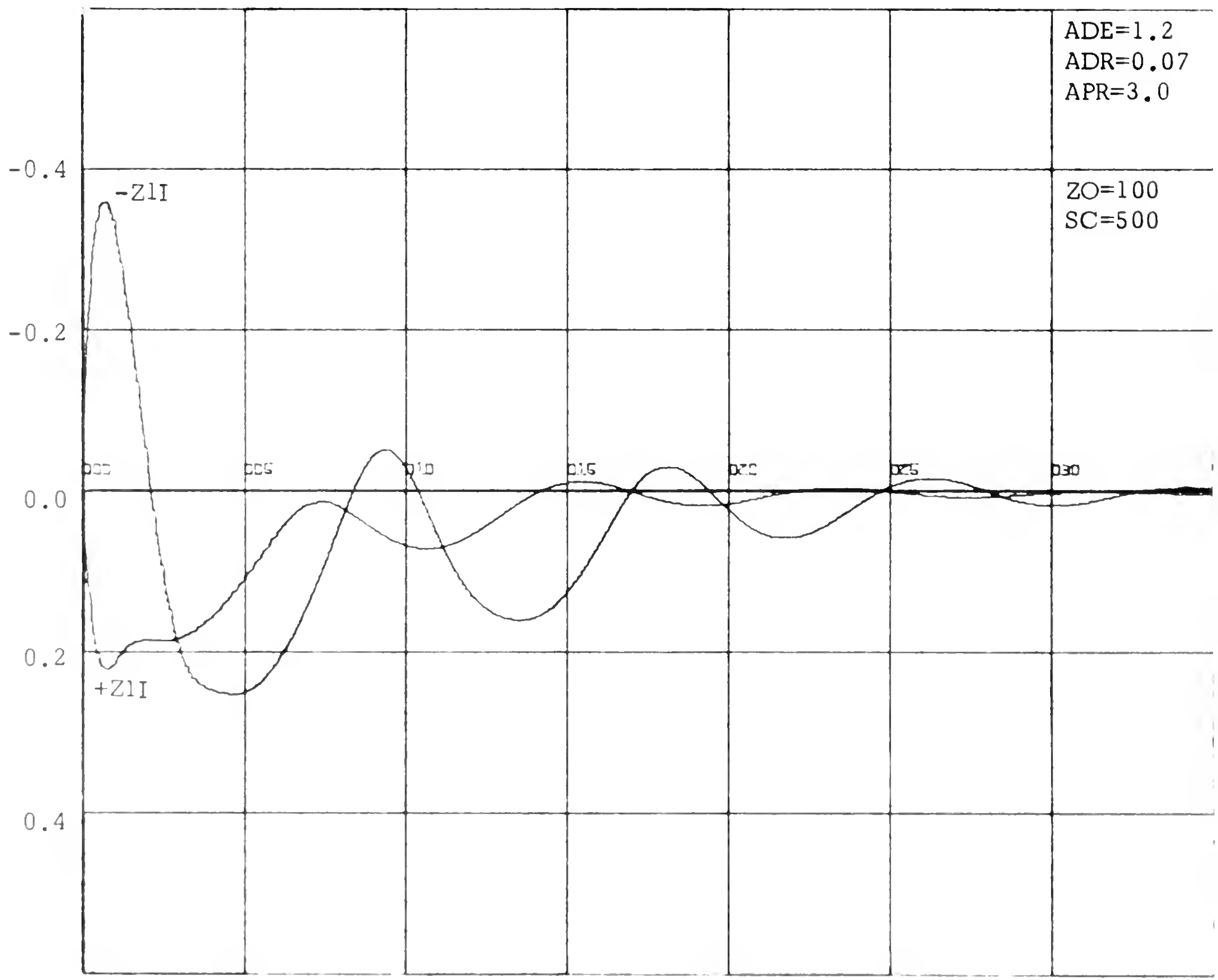
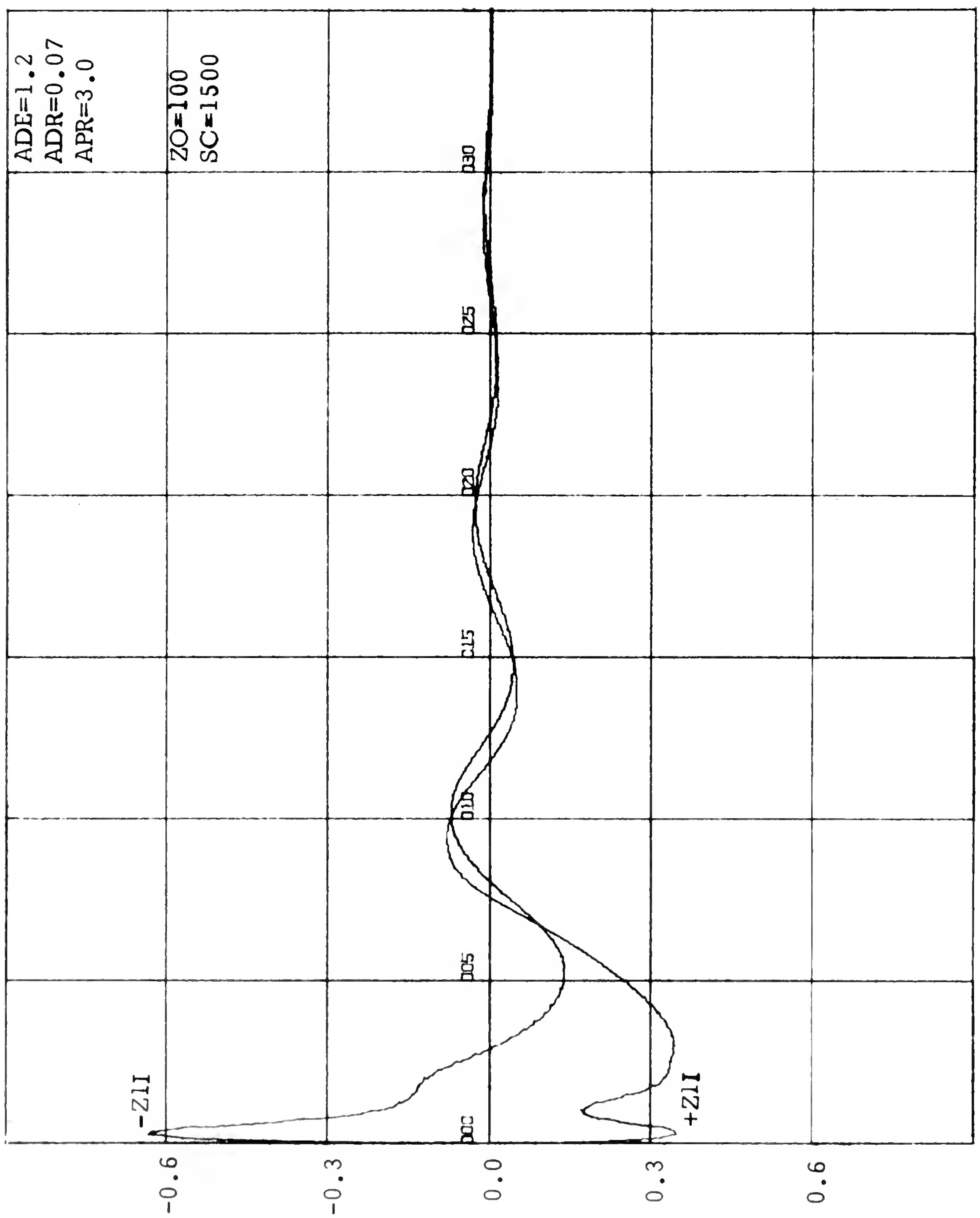
-0.3
0.0
0.3
0.6



ADE=1.2
ADR=0.07
APR=3.0
ZO=100
SC=500
-Z1I
+Z1I
-0.4
-0.2
0.0
0.2
0.4



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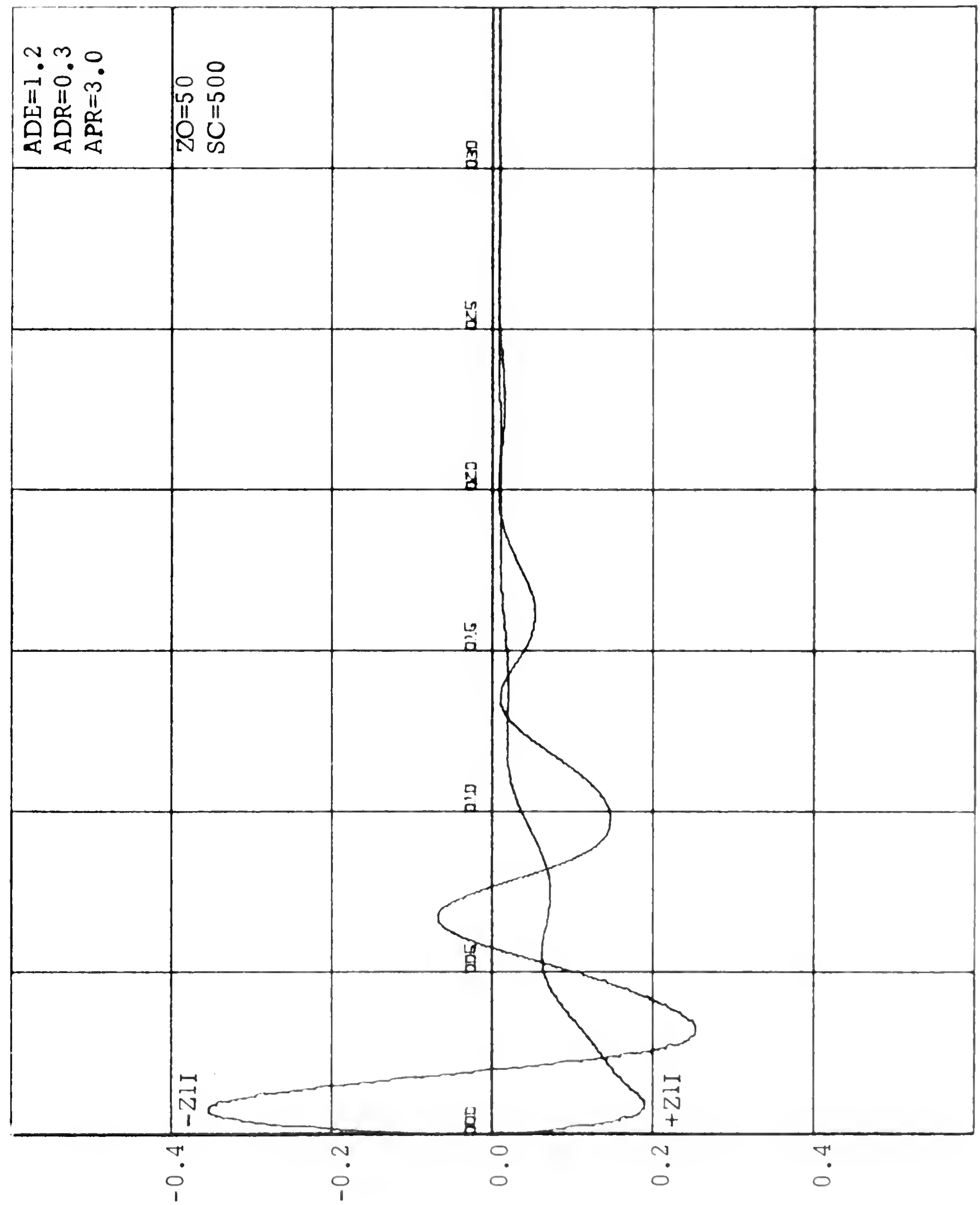
ADR=0.07
APR=3.0
ZO=100
SC=1500
-0.6
-0.3
0.0
0.3
0.6


ADE=1.2
ADR=0.3
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ZO=50
SC=500
-Z11
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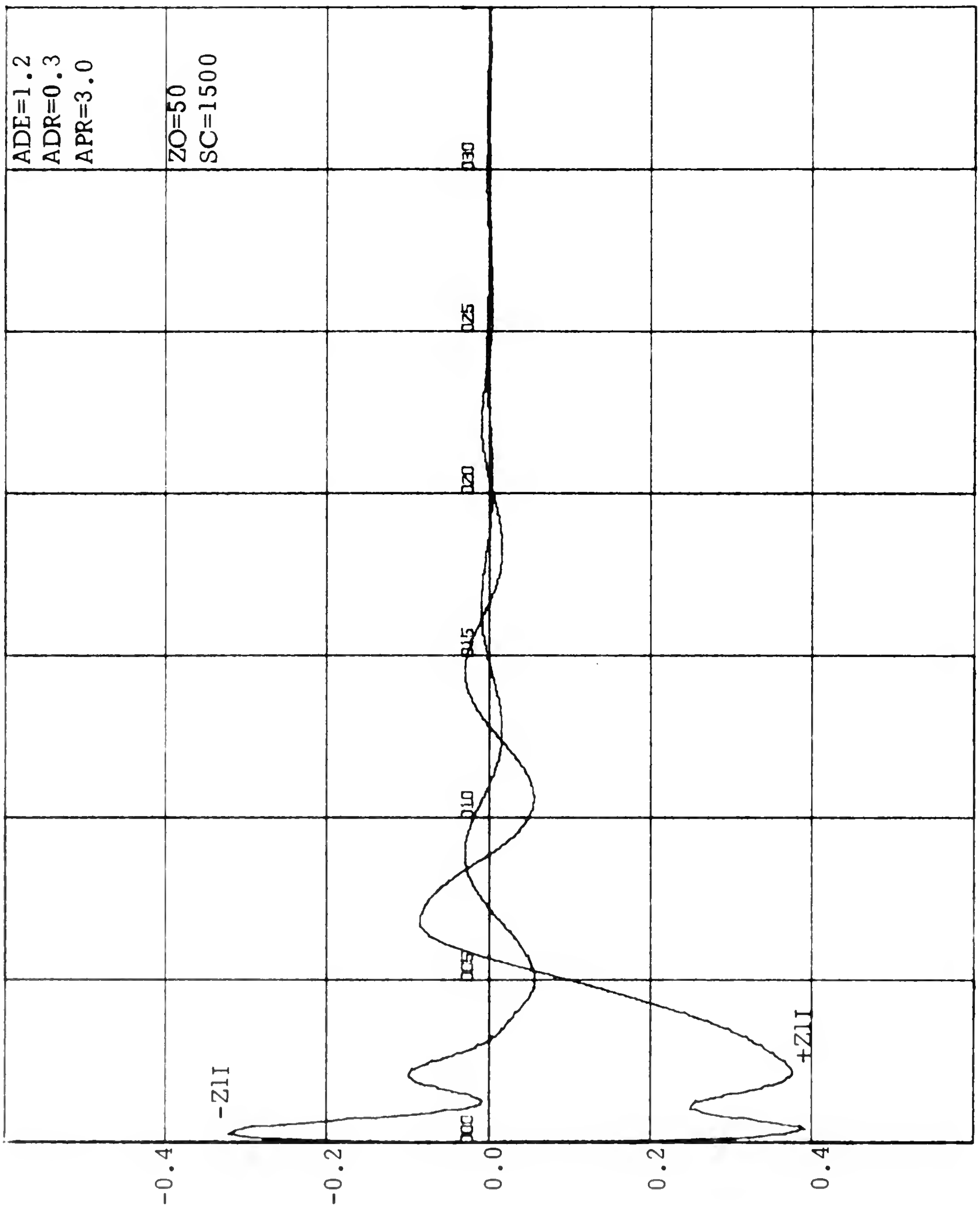
-0.2
0.0
0.2
0.4



ADE=1.2
ADR=0.3
APR=3.0
ZO=50
SC=1500
-Z1I
+Z1I
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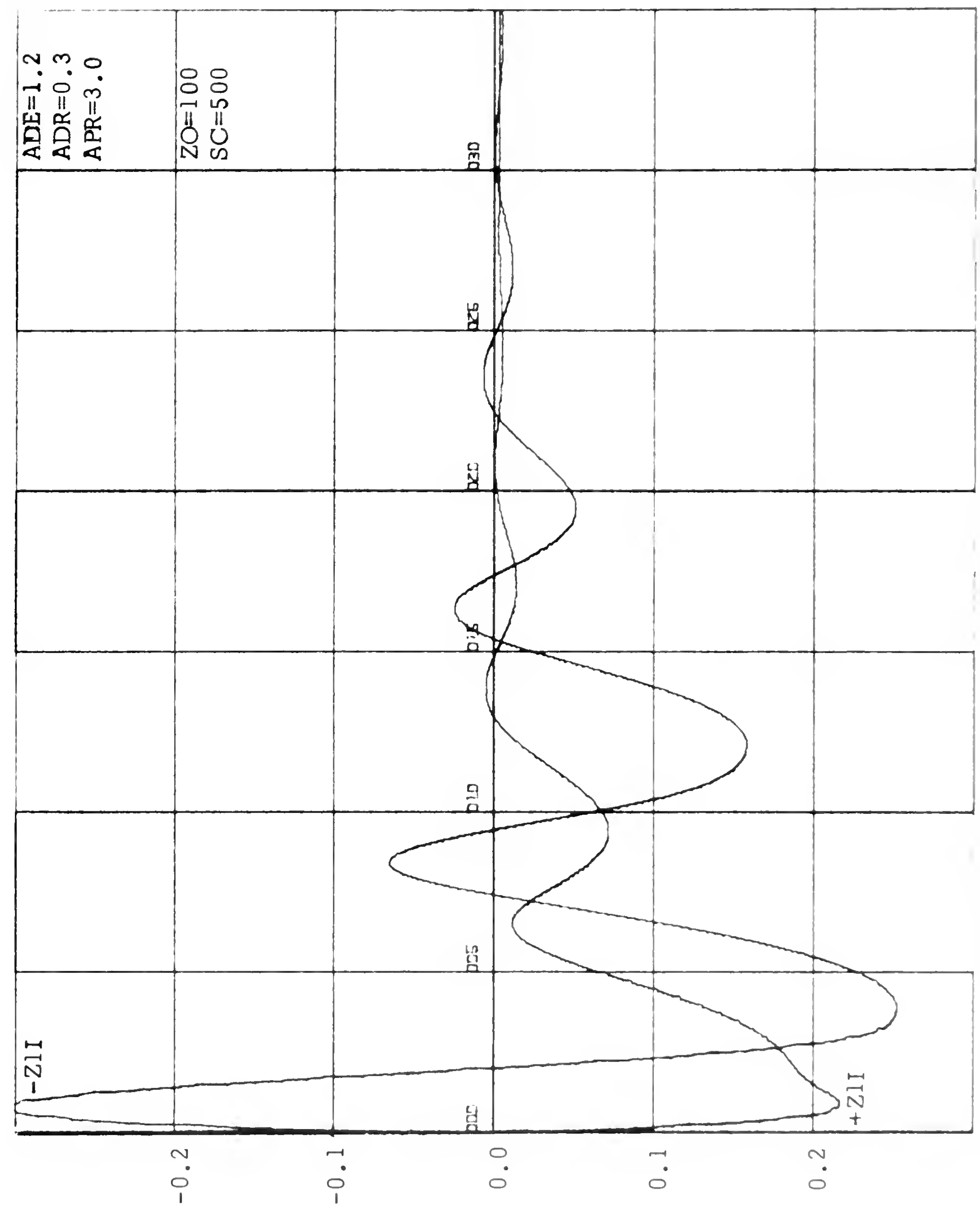
-0.2
0.0
0.2
0.4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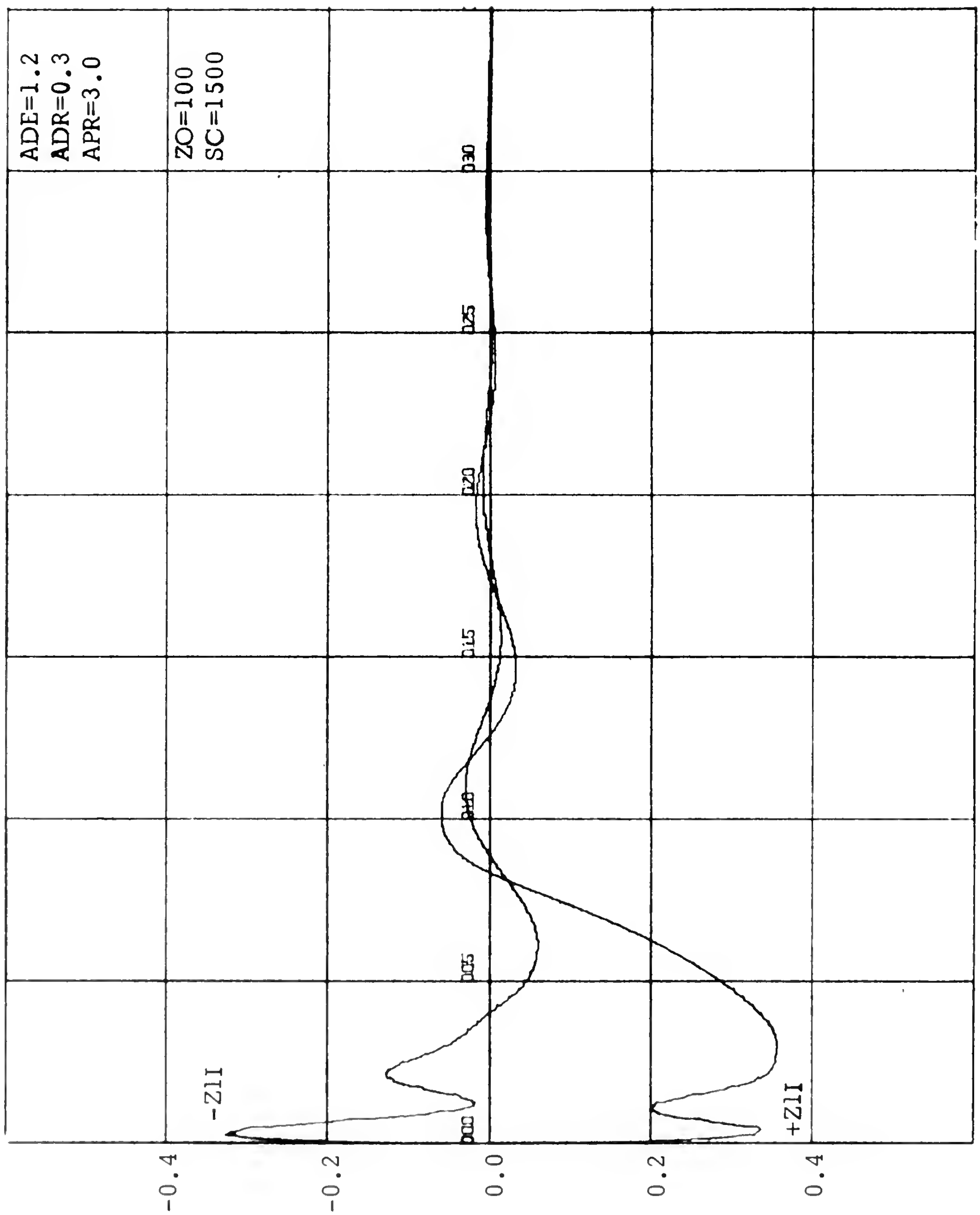
0.2



ADE=1.2
ADR=0.3
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ZO=100
SC=1500
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+Z11
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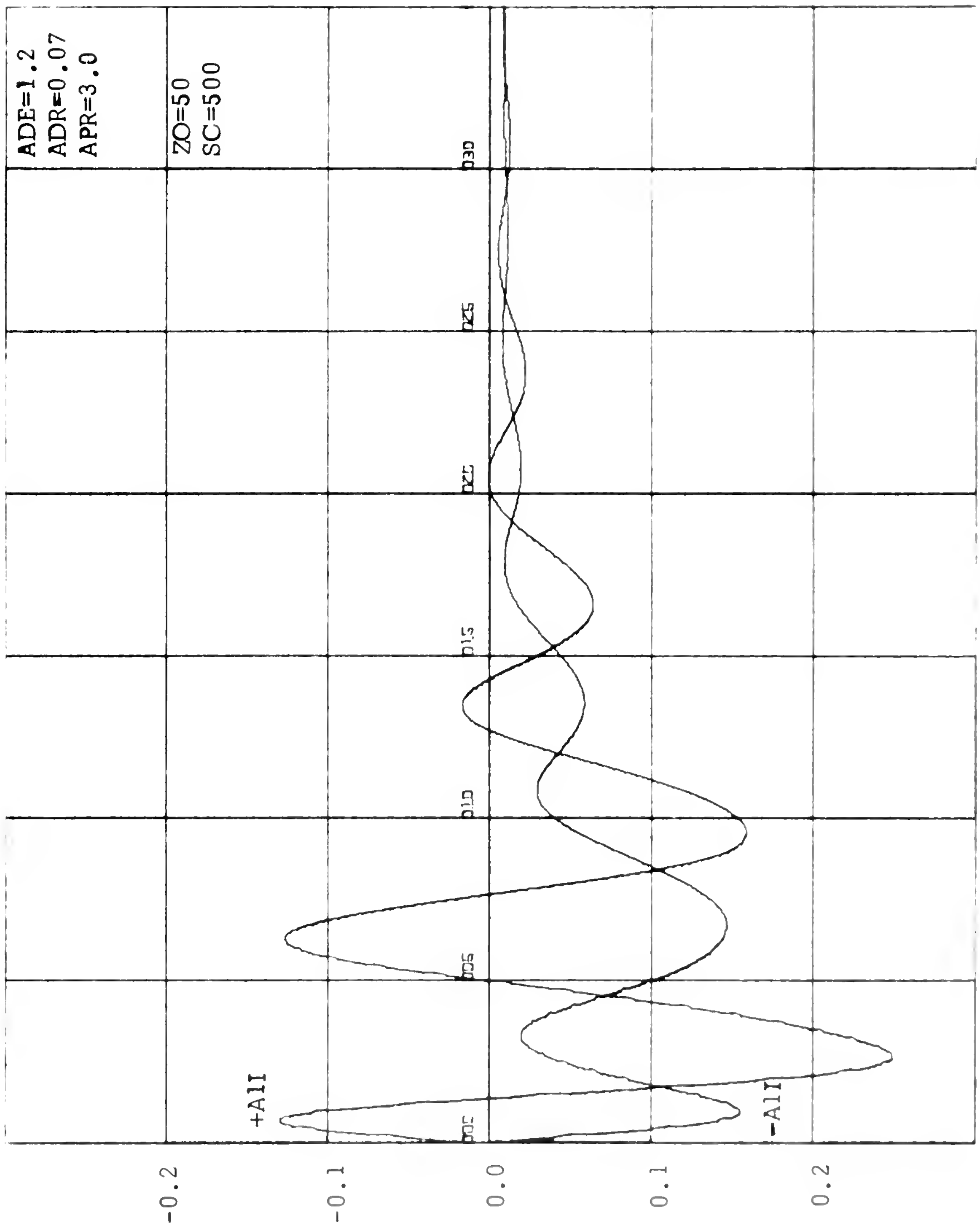
-0.2
0.0
0.2
0.4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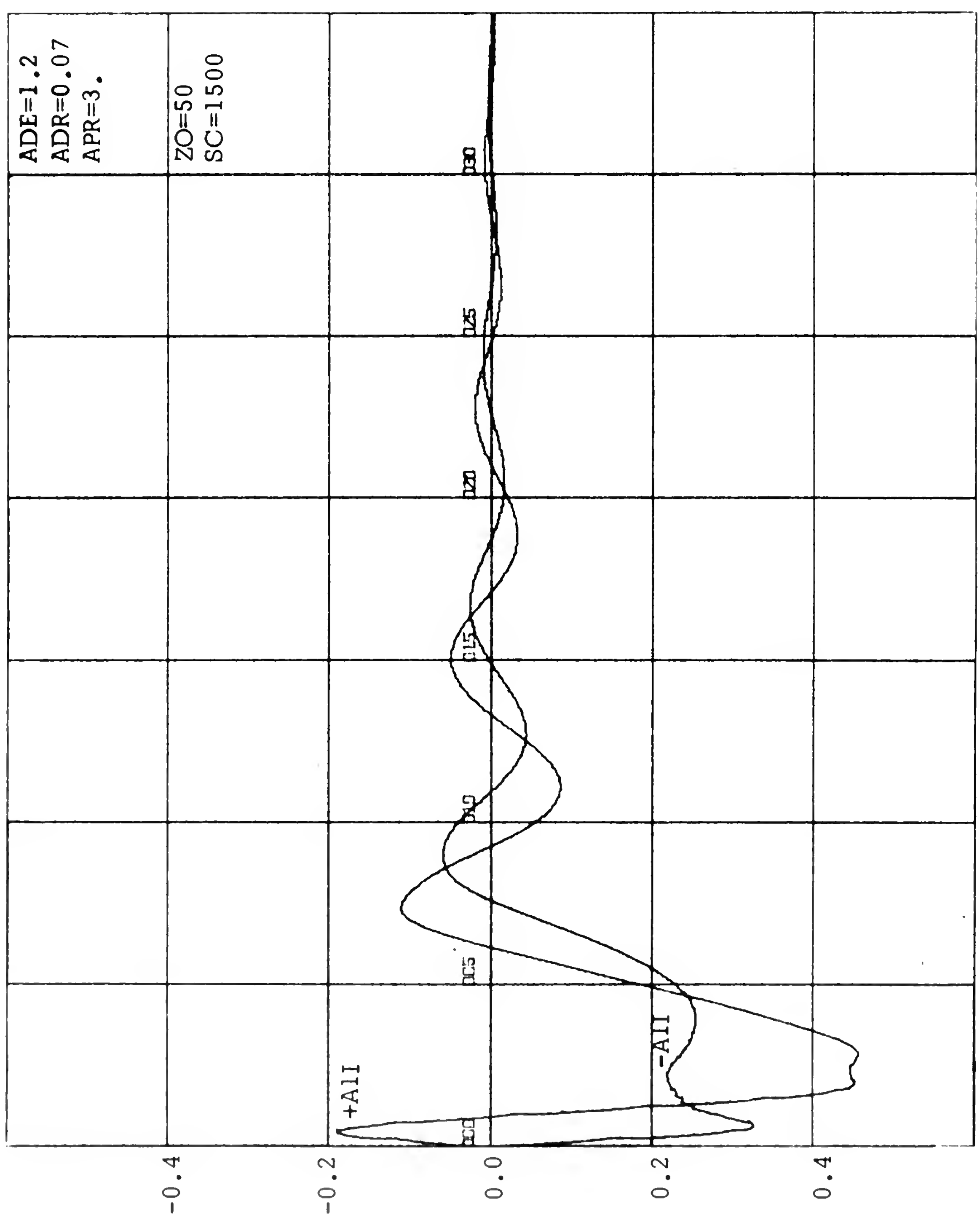
APR=3.0
ZO=50
SC=500
+AlI
-AlI
000
005
010
015
020
025
030
-0.2
-0.1
0.0
0.1
0.2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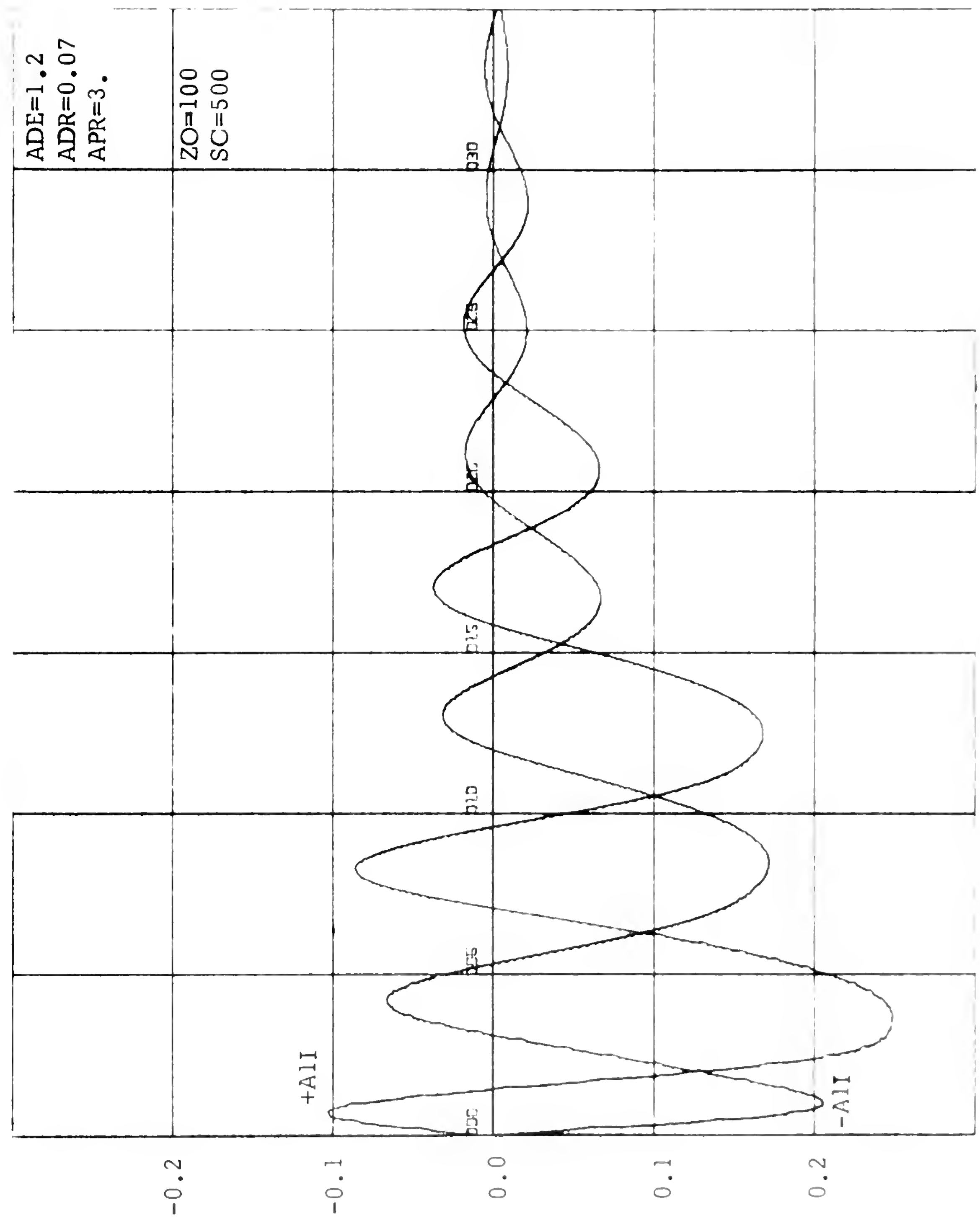
-0.2
0.0
0.2
0.4



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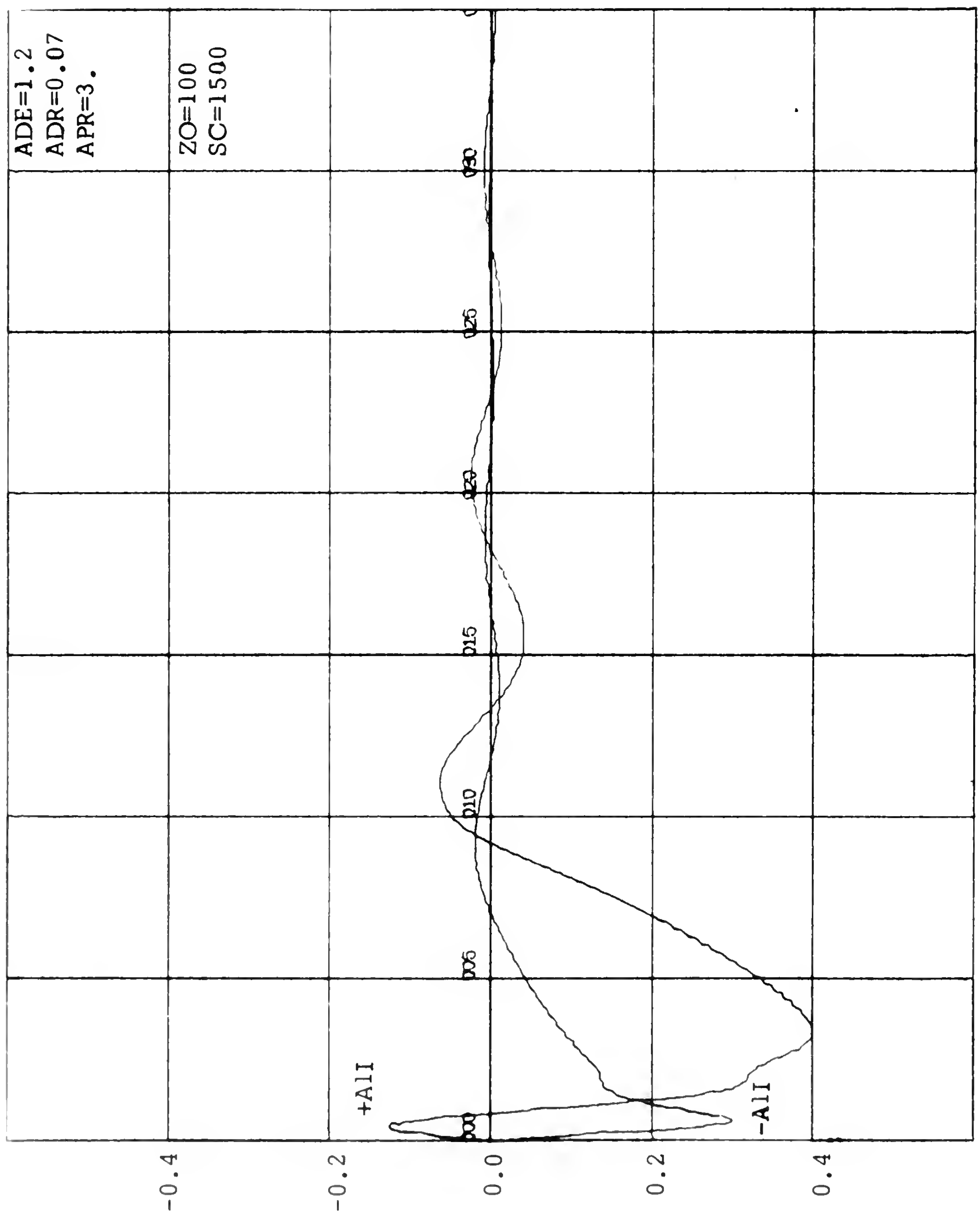
0.1
0.2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015
010
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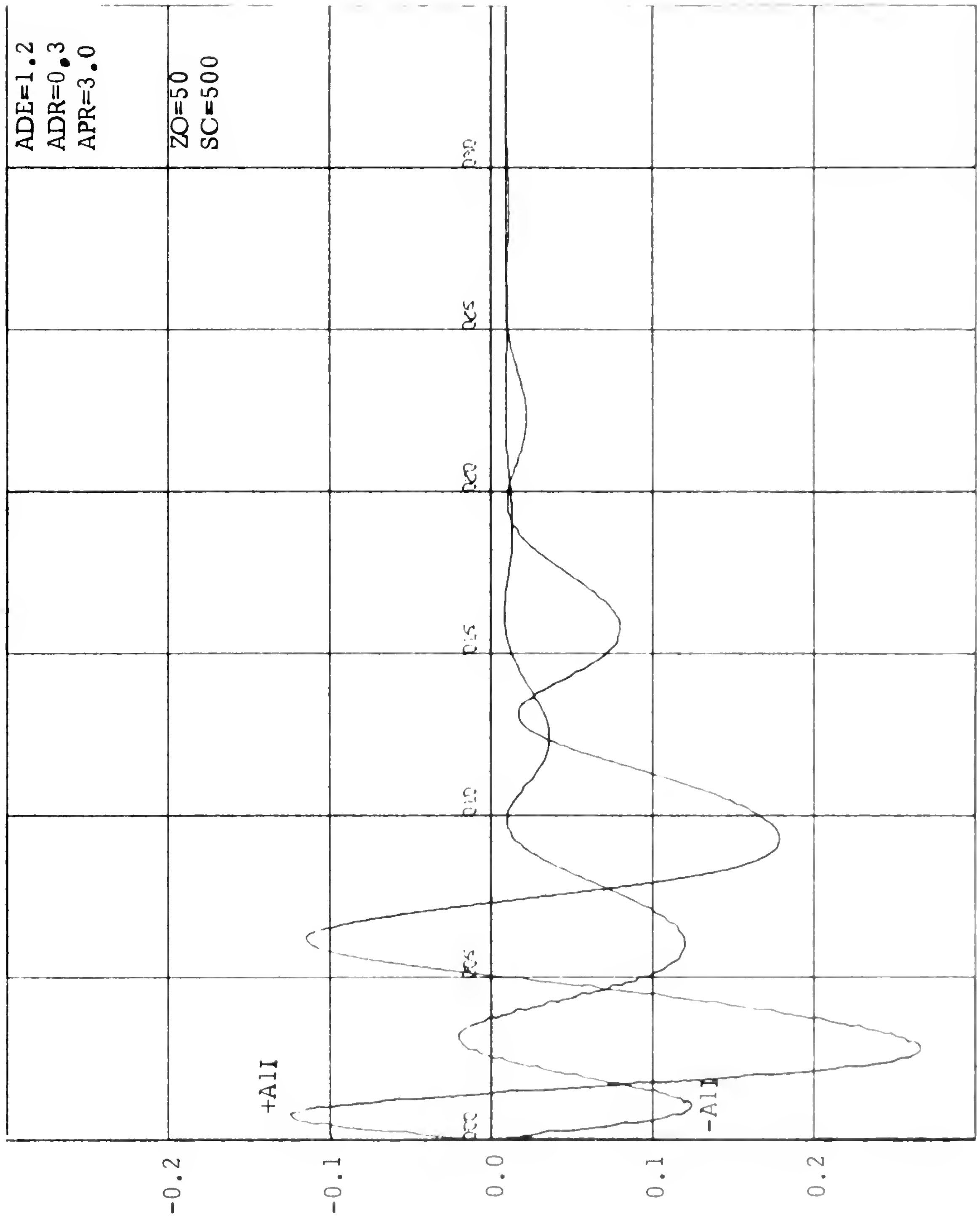
-0.2
0.0
0.2
0.4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ADR=0.3
APR=3.0
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SC=500
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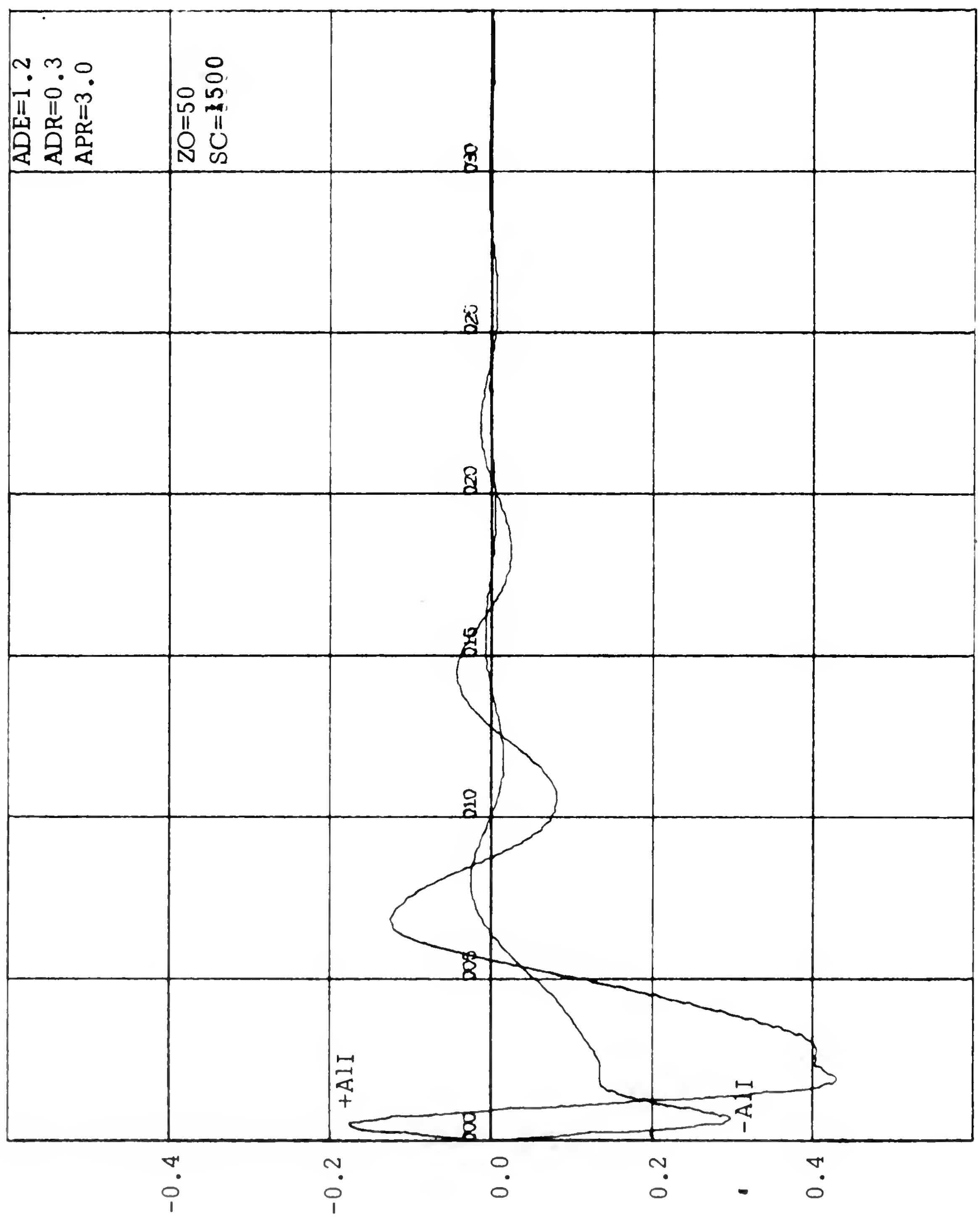
-0.1
0.0
0.1
0.2



ADE=1.2
ADR=0.3
APR=3.0
ZO=50
SC=1500
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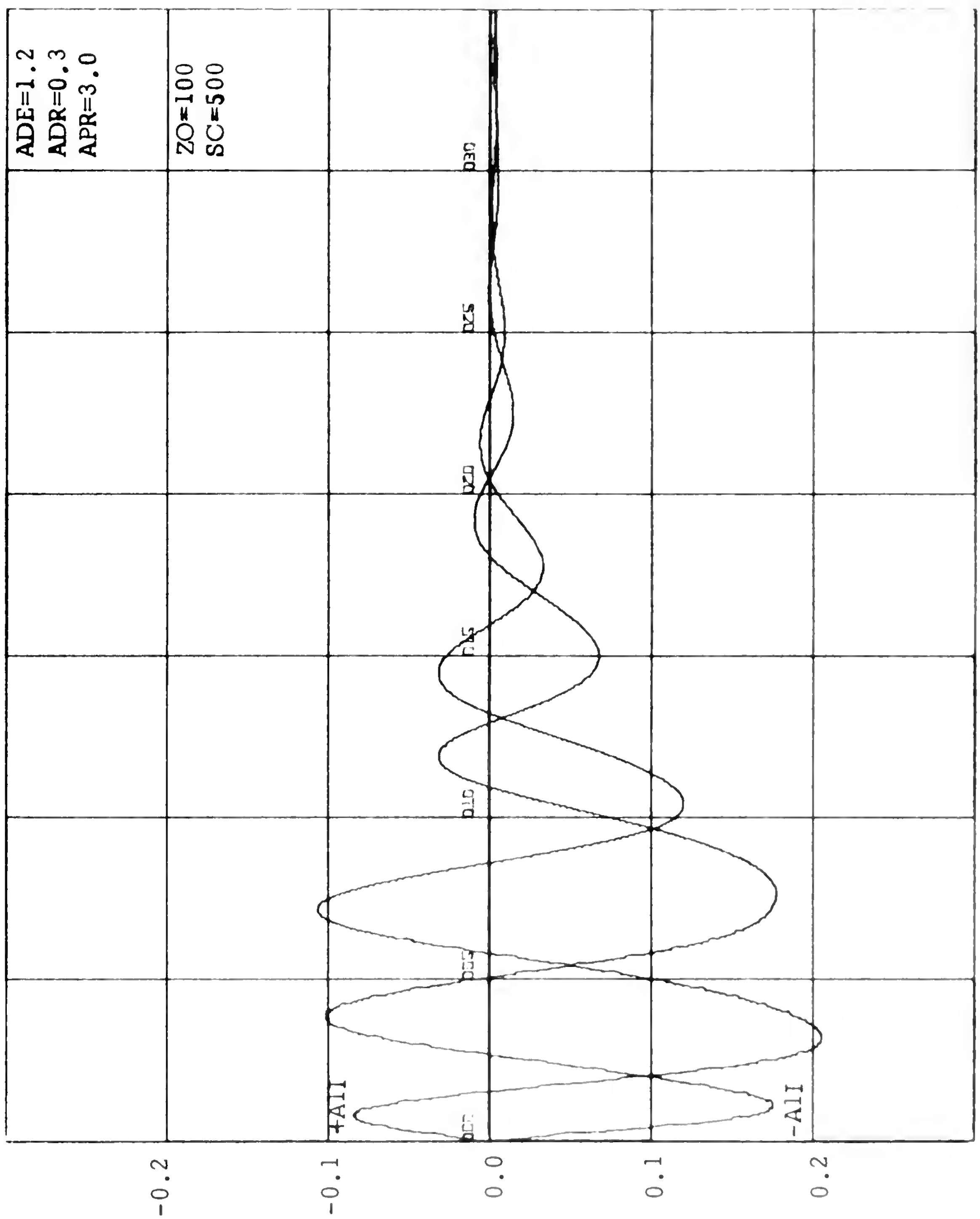
0.2
0.4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ZO=100
SC=500
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-AII
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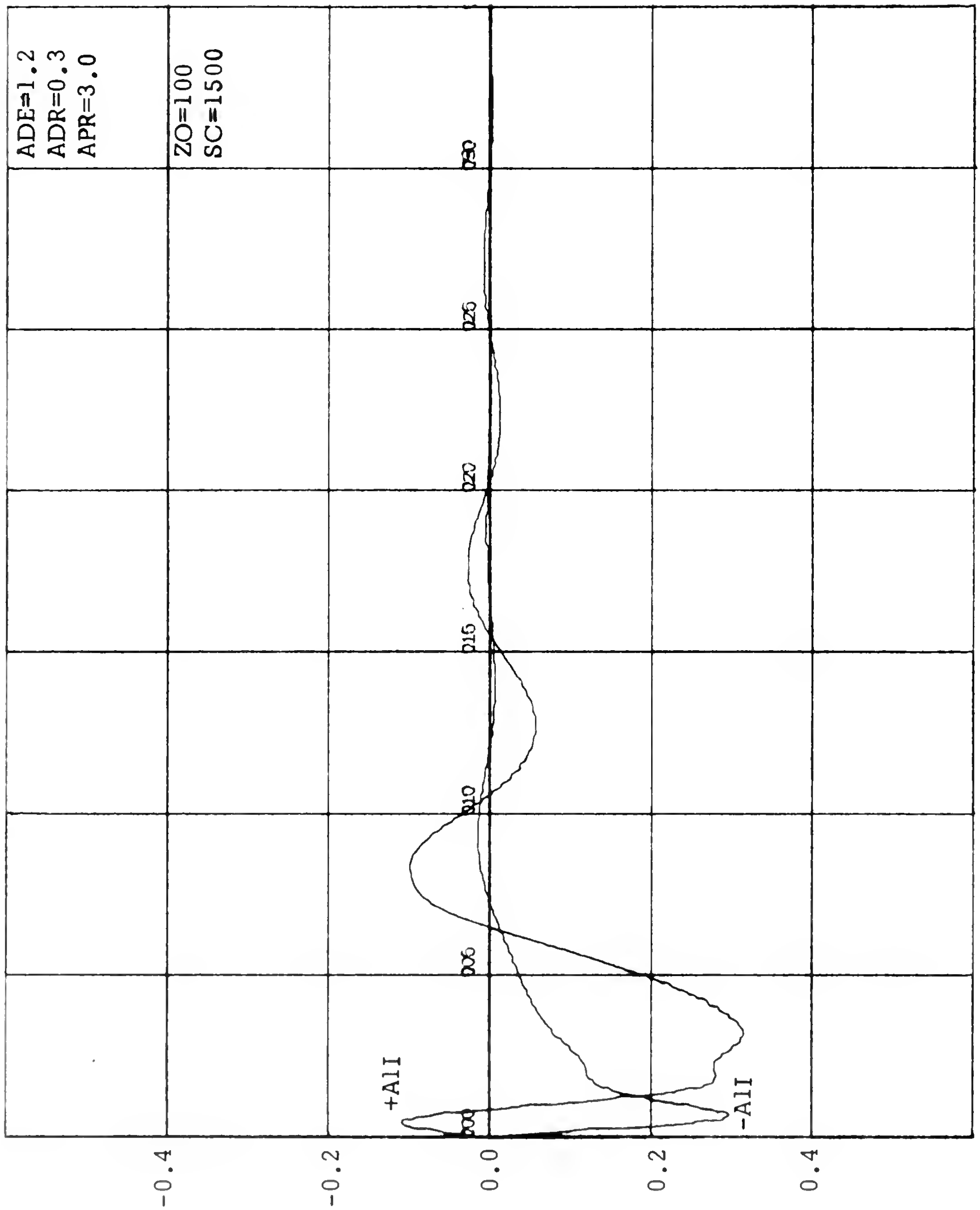
-0.1
0.0
0.1
0.2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APR=3.0
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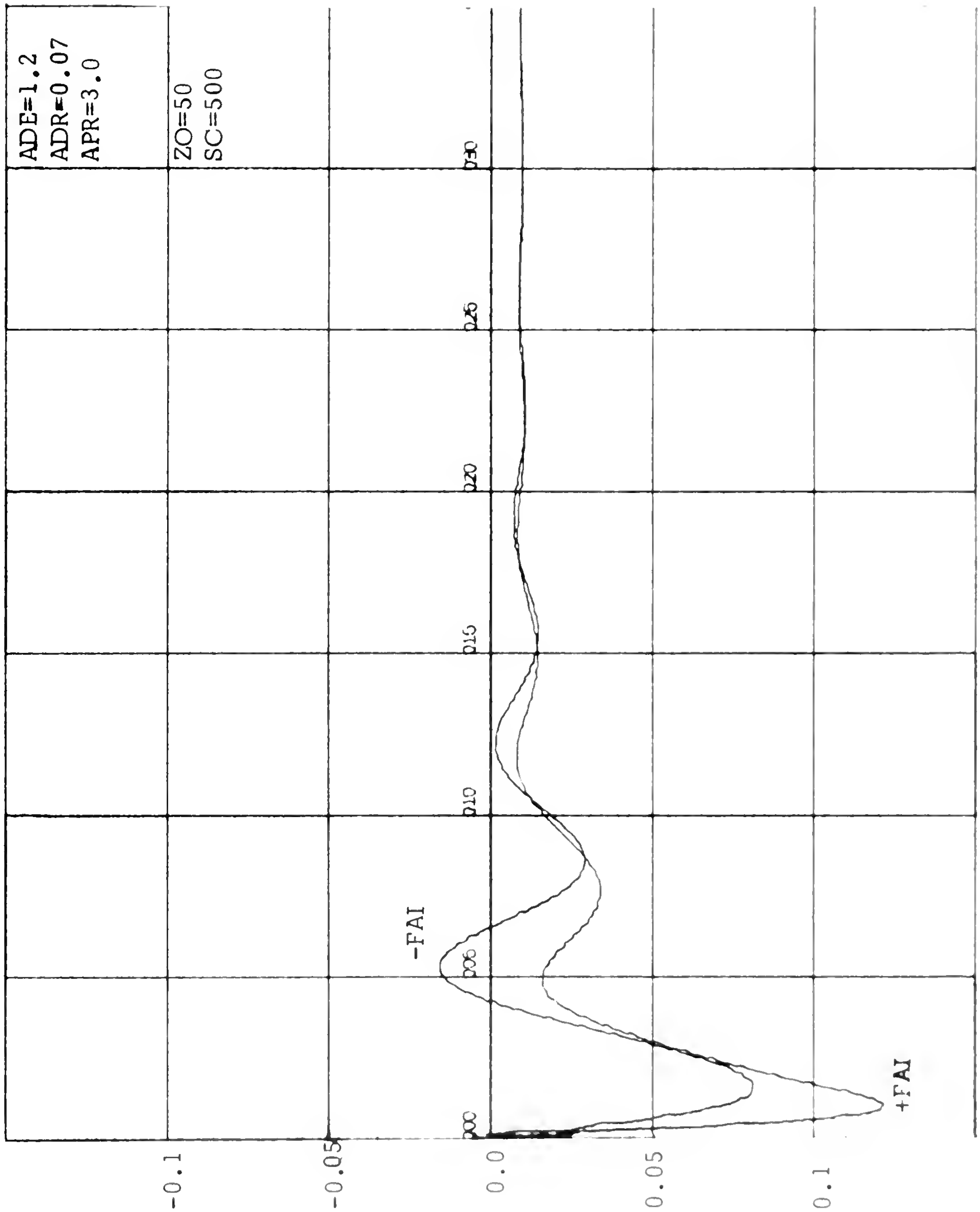
+AlI
-AlI
-0.4
-0.2
0.0
0.2
0.4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APR=3.0
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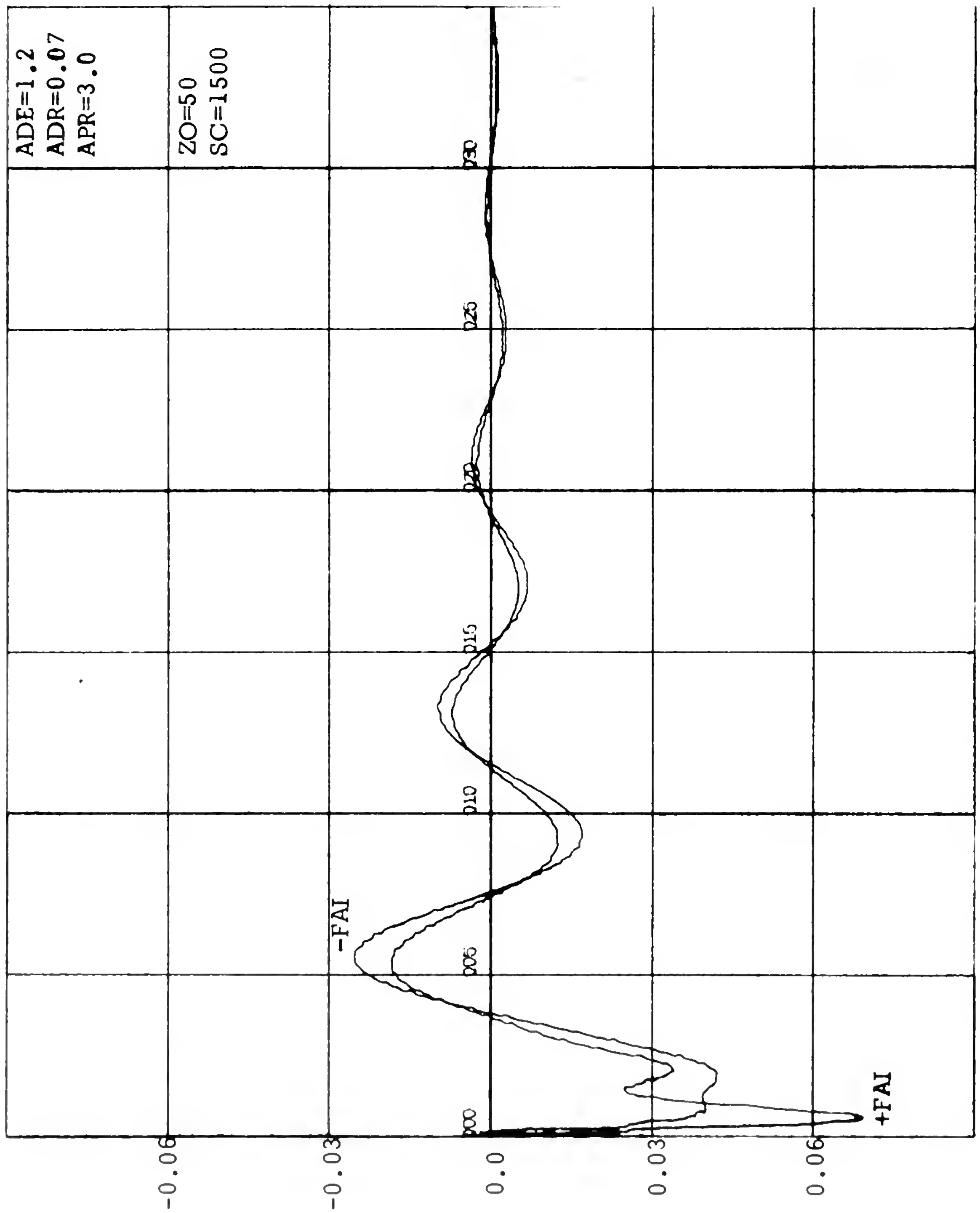
0.05
0.1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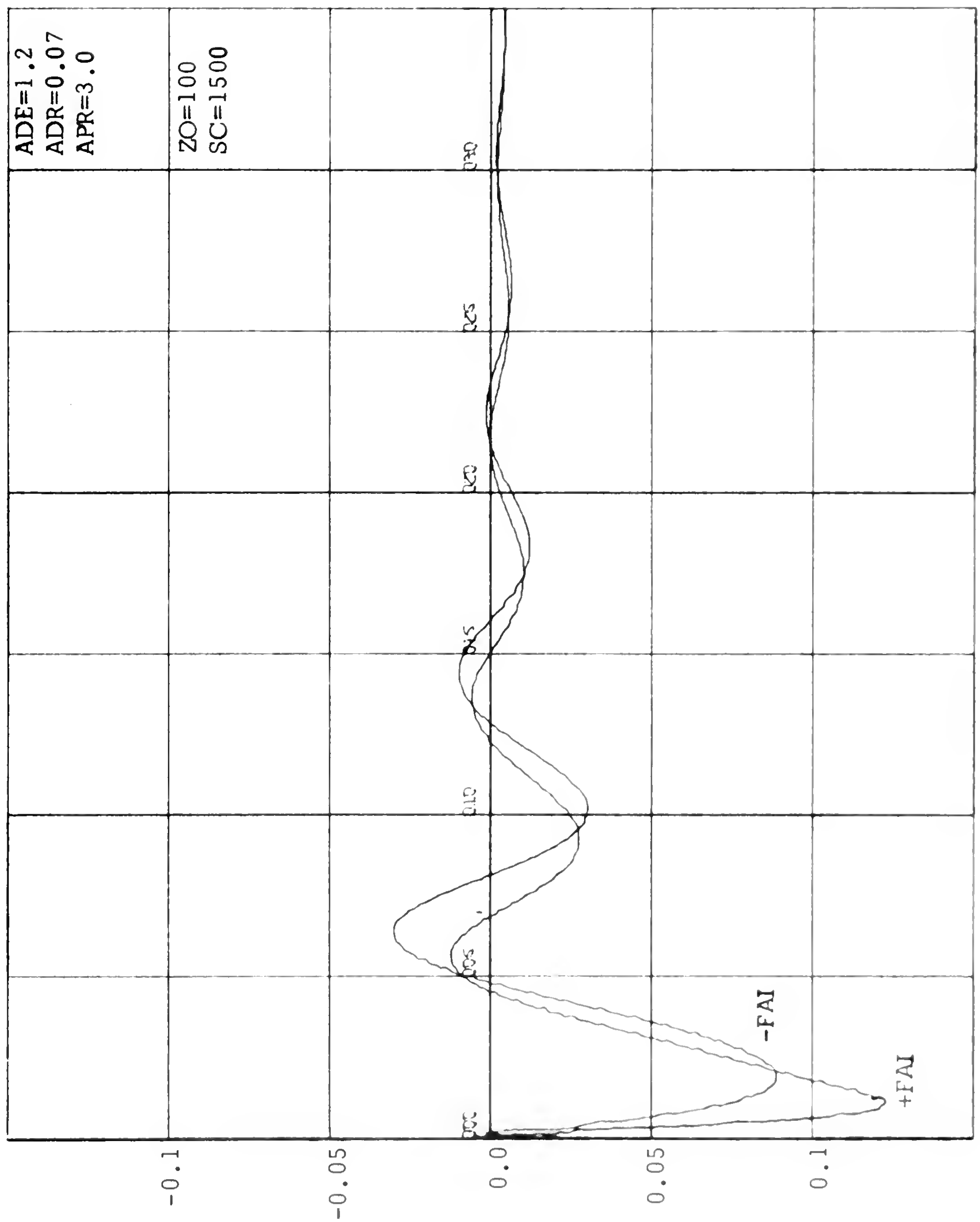
0.03
0.06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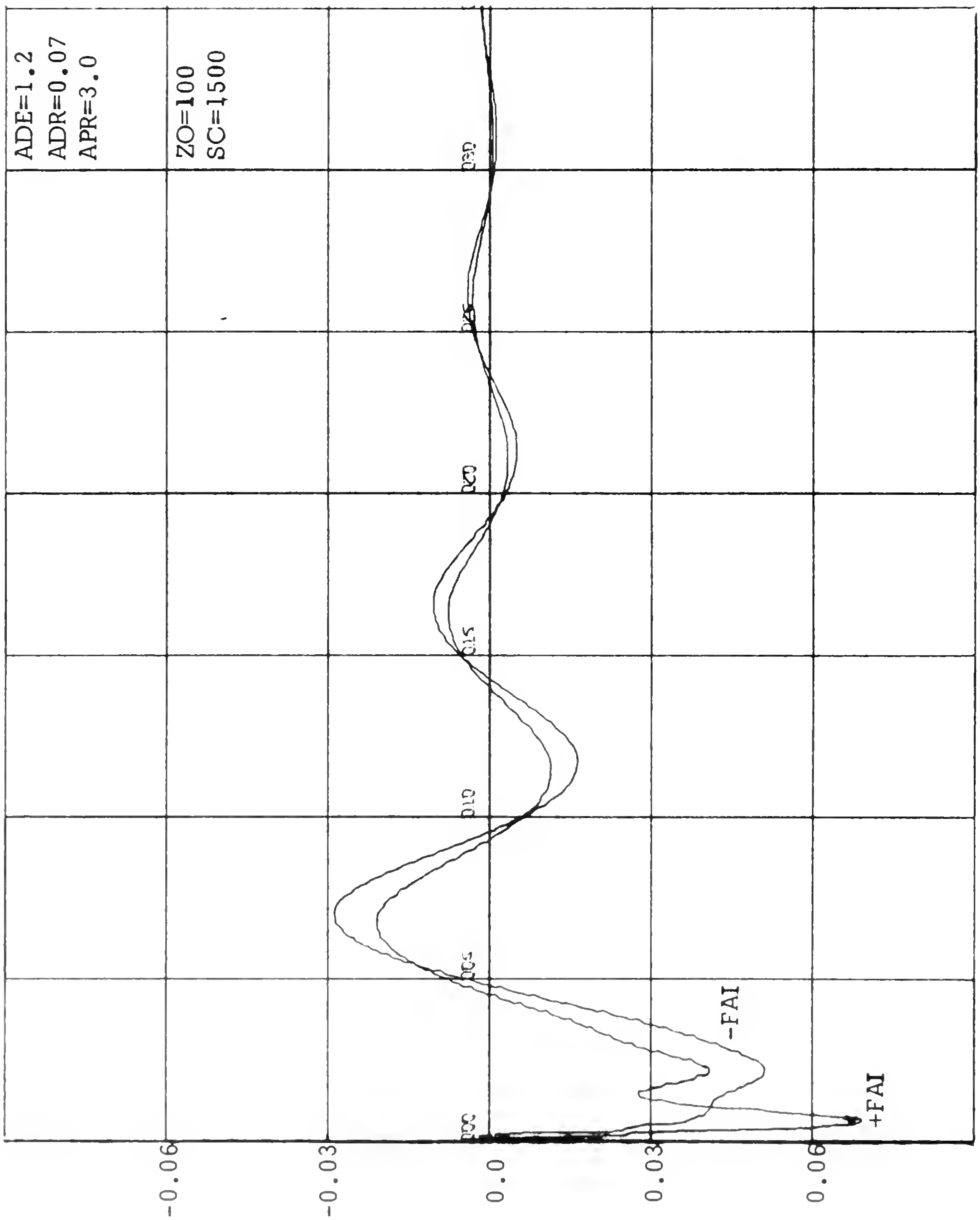
0.0
0.05
0.1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+FAI
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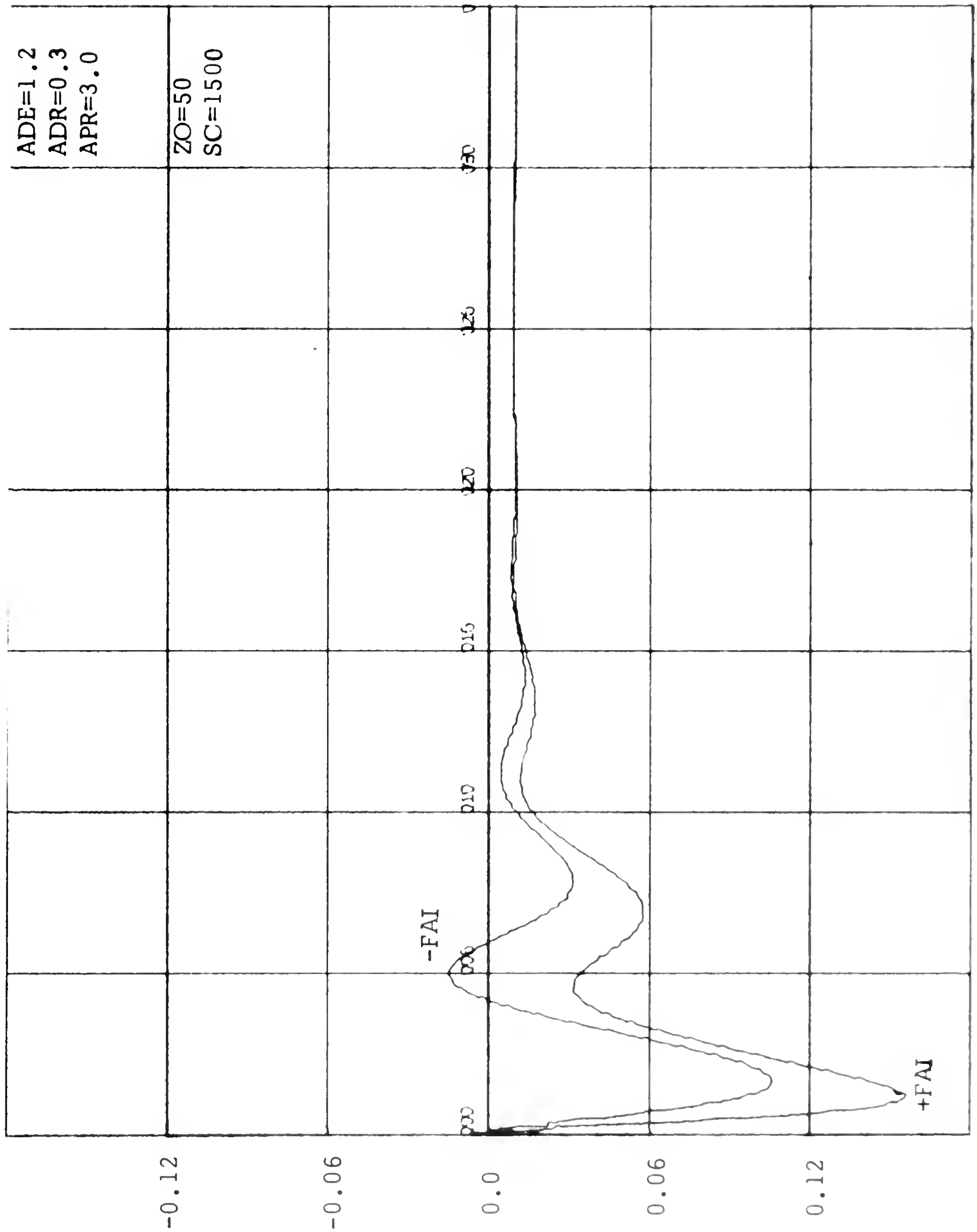
-0.03
0.0
0.03
0.06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ADR=0.3
APR=3.0
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+FAI
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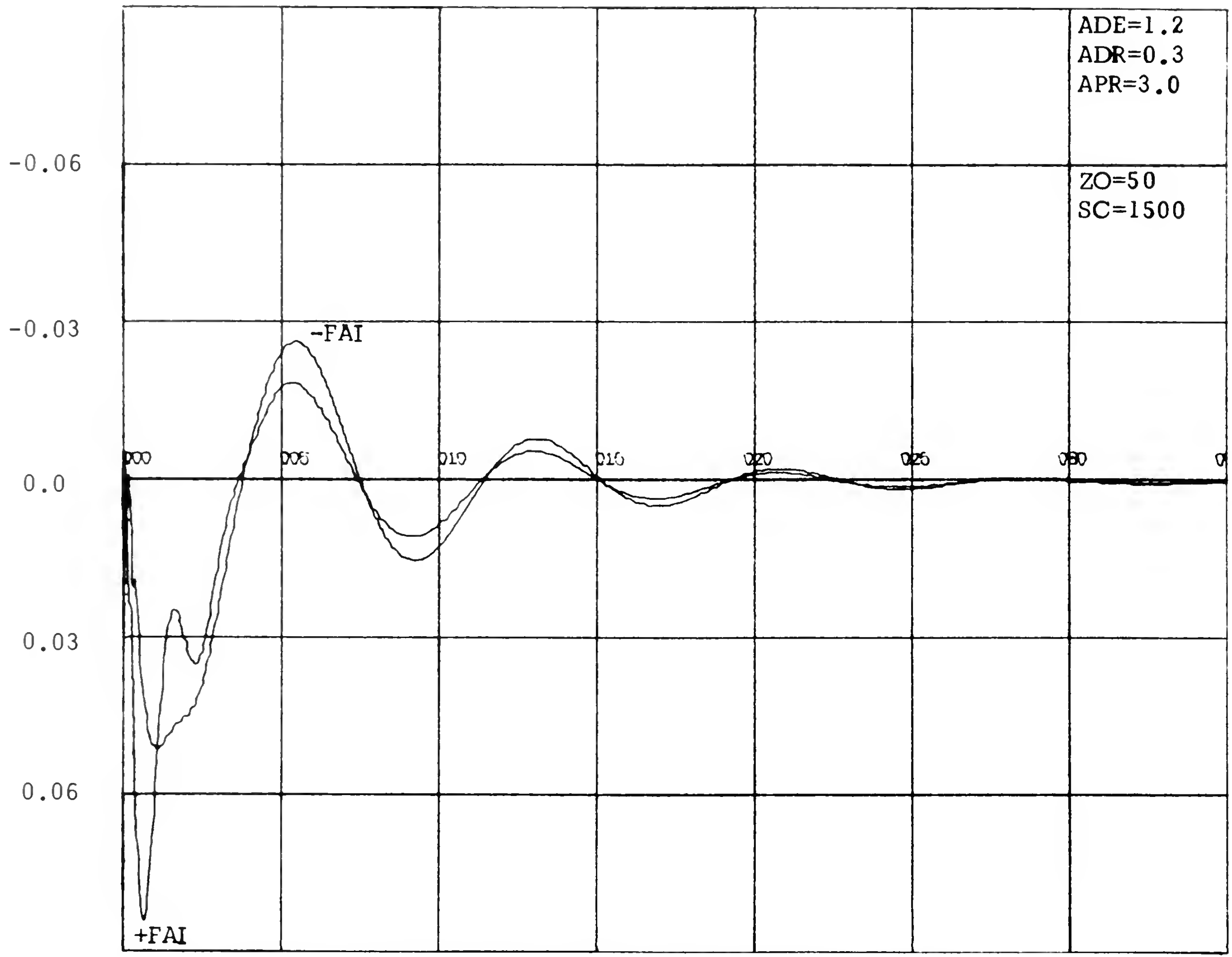
-0.06
0.0
0.06
0.12


ADE=1.2
ADR=0.3
APR=3.0
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SC=1500
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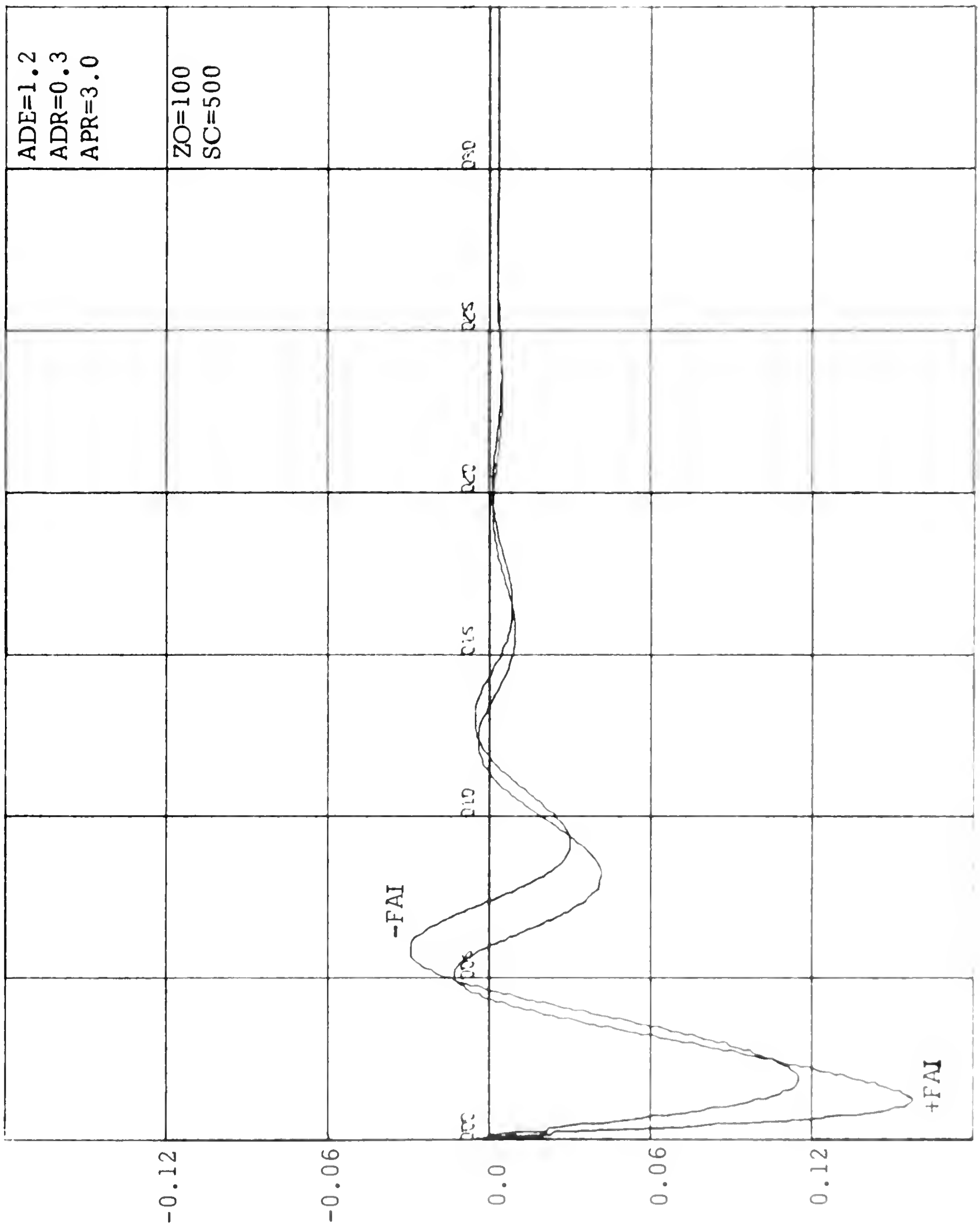
+FAI
-0.06
-0.03
0.0
0.03
0.06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ADR=0.3
APR=3.0
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+FAI
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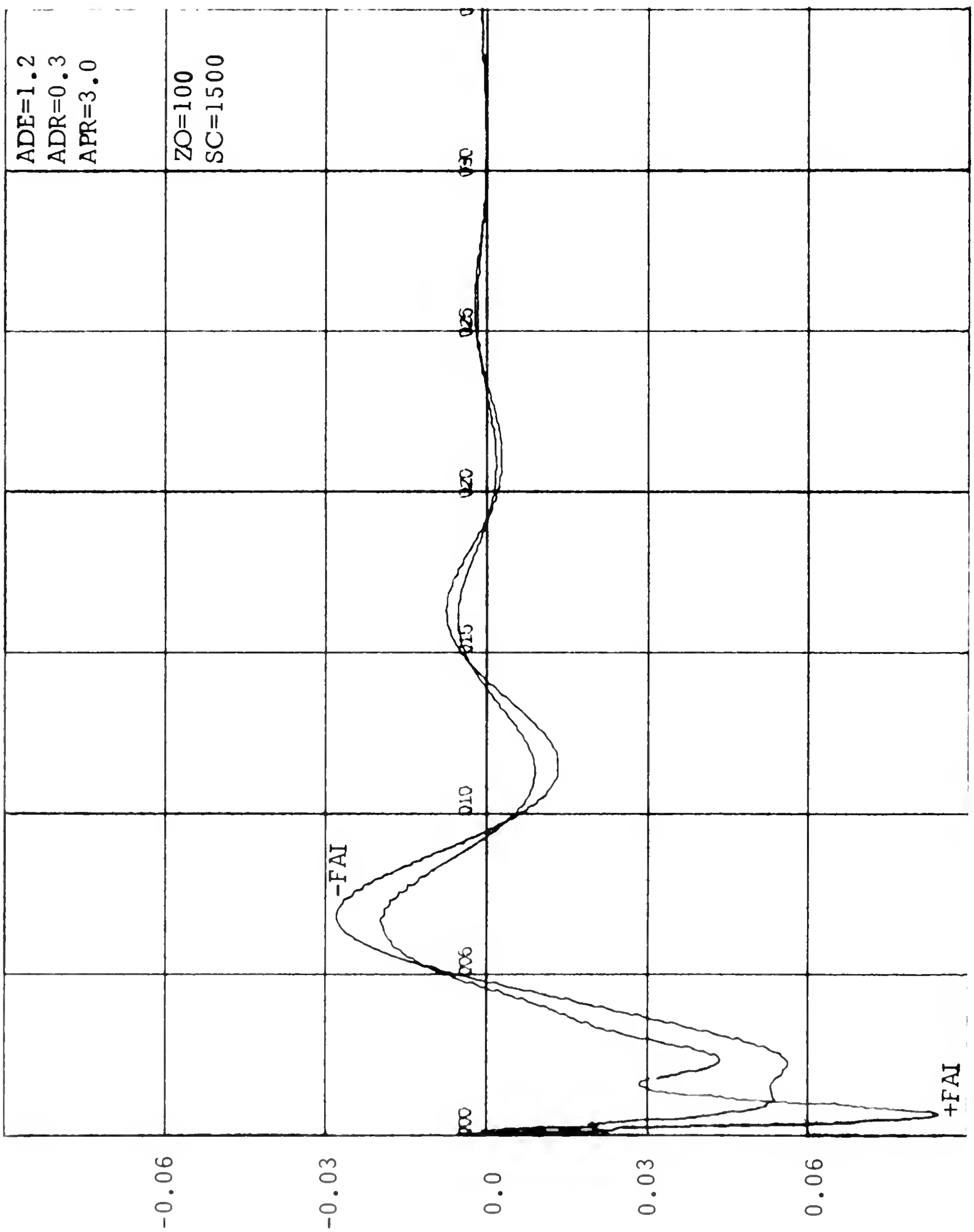
0.0
0.06
0.12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ADR=0.3
APR=3.0
ZO=100
SC=1500
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+FAI
-0.06
-0.03
0.0
0.03
0.06

APPENDIX B

SYMBOLS AND ABBREVIATIONS USED IN THE COMPUTER PROGRAMS

Z	vertical position in ft. (initial condition ZI)
Z1	vertical velocity in ft/sec. (initial condition Z1I)
Z2	vertical acceleration in ft/sec^2
FV	summation of vertical forces
F1	summation of the vertical forces less the vertical tension component
X1	horizontal velocity in ft/sec. (initial condition X1I)
X2	horizontal acceleration in ft/sec^2
FH	summation of the horizontal forces
F2	summation of the horizontal forces less the horizontal tension force
FF	the absolute value of F1
FG	the absolute value of F2
FE	the angle with which the tension force acts on the body in radians
SFE	the sine of FE
CFE	the cosine of FE
TSF	the vertical component of the tension force
TCF	the horizontal component of the tension force
C	horizontal velocity in knots
VAN	the angle of the velocity vector relative to the horizontal in radians
V2	the magnitude of the velocity vector squared

A1	the body angle of attack relative to the horizontal in radians (initial condition A1I)
Y1	the body pitch rate about the Y axis in radians/sec (initial condition Y1I)
Y2	angular acceleration of the body about the Y axis in radians/second^2
YM	summation of the moments on the body about the Y axis
AR	the body angle of attack relative to the velocity vector in radians
A	AR in degrees
FA	the fin angle of attack relative to the horizontal in radians (initial condition FAI)
FB	used in the fin servo loop
FC	used in the fin servo loop
FD	summation of the inputs and the feedback in the fin servo loop
BR	the fin angle of attack relative to the velocity vector in radians
B	BR in degrees
CL	coefficient of lift of the body
CD	coefficient of drag of the body
CM	coefficient of moment of the body
T	the tension force
CV	the cosine of VAN
SV	the sine of VAN
ZAB	the absolute value of z
AD	the antenna drag

BL the body lift

BD the body drag

YCM the moment on the body about the Y axis

YADS the moment on the body due to the vertical component of the antenna drag

YADC the moment on the body due to the horizontal component of the antenna drag

ZD in the first program this is used to measure the relative depth error from 4.0 ft; in the second program it is used to compute the antenna drag

DEE the depth error in psi/ft

DE DEE times the coefficient ADE

DRR the depth rate in psi·sec/ft

DR DRR times the coefficient ADR

YPR the body pitch rate (Y1) times the coefficient APR

ADE the depth error coefficient

ADR the depth rate coefficient

APR the pitch rate coefficient

THE FOLLOWING ARE USED SOLELY IN THE SECOND COMPUTER PROGRAM IN THE REALIZATION OF THE NEW TENSION FORCE

FSS the steady-state value of FE

ZO the vertical approximation to the dynamic length of the cable; see Fig. 17

SC the spring-constant value of the cable in pounds/ft

SFS the sine of FSS

LO the approximation to the dynamic length of the cable; see Fig. 17

TFS	the tangent of FSS
XO	the horizontal approximation to the dynamic length of the cable; see Fig. 17
ZN	the new computed value of ZO
XR	the relative movement out of steady state in the X direction
X	the change in position out of the steady state in the X direction
XN	the new computed value of XO
LN	the new computed length of the dynamic length of the cable
L	the change in length of the cable
FE	as stated previously, but computed differently

```
TITLE                  INITIAL  BUOY  MODEL

***    PARAMETERS TO BE STUDIED AND INITIAL CONDITIONS
PARAM ADE=1.2,ADR=.300,APR=0.0
INCON ZI=4.0,Z1I=1.E-12,X1I=20.2536,A1I=.10472,Y1I=1.E-12,FAI=.02,...
      FBI=1.E-12

***    VERTICAL FORCE, ACCELERATION, VELOCITY, AND POSITION

       Z=INTGRL(ZI,Z1)
       Z1=INTGRL(Z1I,Z2)
       Z2=FV/70.
       FV=F1+TSF
       F1=-BL*CV-BD*SV-AD*SV

***    HORIZONTAL FORCE, ACCELERATION, AND VELOCITY

       X1=INTGRL(X1I,X2)
       X2=FH/70.
       FH=F2+TCF
       F2=BL*SV-BD*CV-AD*CV

***    CALCULATION OF THE ANGLE FE
       FF=ABS(F1)
       FG=ABS(F2)
       FE=ATAN2(FF,FG)
       SFE=SIN(FE)
       CFE=COS(FE)
***    HORIZONTAL AND VERTICAL COMPONENTS OF TENSION
       TCF=CFE*T
       TSF=SFE*T
***    VELOCITY ANGLE AND MAGNITUDE SQUARED
       VAN=ATAN2(Z1,X1)
       V2=Z1**2+X1**2
       CV=COS(VAN)
       SV=SIN(VAN)

***    ANGULAR MOMENT, ACCELERATION, VELOCITY, AND POSITION ABOUT Y AXIS

       A1=INTGRL(A1I,Y1)
       Y1=INTGRL(Y1I,Y2)
       Y2=YM/975.
       YM=YCM+YADC-YADS
```

```
***    FIN SERVO LOOP

       FA=INTGRL(FAI,FB)
       FB=INTGRL(FBI,FC)
       FC=FD*1000.
       FD=-.0443*FB-FA-DE+DR+YPR
***    COMPUTATION OF ANGLES OF ATTACK RELATIVE TO THE VELOCITY VECTOR
       AR=A1+VAN
       A=AR*57.29578
       BR=FA+VAN
       B=BR*57.29578
       C=X1/1.6878

***    CALLING THE BODY COEFFICIENTS AND TENSION FORCE OUT OF THE SUBROUTINE
       CL,CD,CM,T=TINK(A,B,C)
***    COMPUTING ANTENNA DRAG
       ZAB=ABS(Z)
       AD=ZAB*.04146*V2
***    COMPUTING BODY LIFT, DRAG, AND MOMENT
       BL=CL*14.17875*V2
       BD=CD*14.17875*V2
       YCM=CM*85.0725*V2
***    COMPUTING MOMENTS CAUSED BY THE ANTENNA
       YADS=AD*SV*3.75
       YADC=(ZAB*.5+1.2873)*AD*CV
***    COMPUTING THE THREE INPUTS INTO THE FIN SERVO LOOP
       ZD=4.-Z
       DEE=ZD*.443
       DE=ADE*DEE
       DRR=.443*Z1
       DR=ADR*DRR
       YPR=Y1*APR

***    PROGRAM CONTROL CARDS
CONTRL FINTIM=8.
PRINT  .1,Z,Z1,FV,X1,FH,T,A,B
END
***    AN ADDITIONAL SET OF PARAMETERS TO BE STUDIED
PARAM  ADE=2.5,ADR=.3
END
STOP


***    NOTE--BLANK CARDS AND CARDS MARKED ***, ARE NOT INCLUDED IN ACTUAL PROGRAM
```

FORTRAN

```
      SUBROUTINE TINK(A,B,C,CL,CD,CM,T)
      REAL*8 A,B,C,CL,CD,CM,T,C1,C2
      CL=.0051D 00*B+2.1712552D-01+2.0279223D-02*A+3.1636558D-04*A**2+2.
     1093616D-05*A**3-3.2986561D-07*A**4-8.1116178D-08*A**5
      CD=4.2707372D-02+3.6589646D-03*A+2.5607579D-04*A**2+1.1961132D-05*
     1A**3+2.9840907D-07*A**4-7.5139831D-08*A**5+3.8575977D-09*A**6+2.57
     203807D-10*A**7-2.323927D-11*A**8+(6.8143195D-04+6.6382999D-05*A+1.
     33223814D-06*A**2+1.5116013D-06*A**3-2.1755085D-08*A**4-2.1159367D-
     408*A**5-1.7507392D-12*A**6+8.1546554D-11*A**7+6.089921D-13*A**8)*B
     5+(4.7428856D-05-3.1706617D-06*A+5.6700361D-07*A**2+1.9501584D-07*A
     6**3-1.6379961D-08*A**4-2.6007520-09*A**5+1.8132173D-10*A**6+9.7843
     7775D-12*A**7-6.4075507D-13*A**8)*B**2
      CM=C1+C2
      C1=3.1254762D-02-2.7515873D-03*B-4.8952381D-05*B**2-4.6222222D-06*
     1B**3+(-7.2748931D-03+6.9793037D-06*B+4.9010127D-06*B**2-4.9364676D
     2-07*B**3-8.0377067D-08*B**4)*A+(-6.7932139D-05+5.4210736D-06*B+1.0
     3789731D-06*B**2-9.6995988D-08*B**3-4.6666717D-08*B**4-7.2678165D-0
     49*B**5-4.3351393D-10*B**6)*A**2+(-1.2808628D-05-1.6744241D-06*B+2.
     591426780-07*B**2+2.7469724D-08*B**3-1.4170085D-08*B**4+5.4319291D-
     610*B**5+1.7953234D-10*B**6)*A**3
      C2=(-9.4307248D-07-2.1777229D-08*B+5.4412650D-09*B**2+4.3324391D-0
     19*B**3+4.8010971D-10*B**4-4.7920116D-11*B**5-5.4138508D-12*B**6)*A
     2**4+(3.4636440D-07+3.2453934D-08*B-1.1466209D-08*B**2-1.3877497D-0
     39*B**3+3.9658498D-10*B**4+2.3337991D-11*B**5-1.6875364D-12*B**6)*A
     4**5+(-1.6765721D-08-1.863330D-09*B+7.0670144D-10*B**2+7.5954549D-1
     51*B**3-2.5910672D-11*B**4-1.5201129D-12*B**5+1.0079619D-13*B**6)*A
     6**6
      T=-126.66667D0-62.46296D0*A+.18518712D0*A**2+1.3811717D0*A**3-.205
     176114D0*A**4+7.2587942D-03*A**5+(225.D0+14.51389D0*A-8.1018767D-02
     2*A**2-.16975357D0*A**3+2.443424D-02*A**4-8.5734252D-04*A**5)*C
      RETURN
      END
```

```
TITLE                 STUDY OF   T = 1940. + L*SC

***    PARAMETERS TO BE STUDIED AND INITIAL CONDITIONS
PARAM  ADE=1.2,ADR=.30,APR=3.,FSS=1.3307,ZD=100.,SC=500.
INCON  ZI=0.0,Z1I=1.E-12,X1I=20.2536,A1I=.08400,Y1I=1.E-12,FAI=1.E-12,...
       FBI=1.E-12,XI=1.E-12

***    VERTICAL FORCE, ACCELERATION, VELOCITY, AND POSITION

       Z=INTGRL(ZI,Z1)
       Z1=INTGRL(Z1I,Z2)
       Z2=FV/70.
       FV=F1+TSF
       F1=-BL*CV-BD*SV-AD*SV

***    HORIZONTAL FORCE, ACCELERATION, AND VELOCITY

       X1=INTGRL(X1I,X2)
       X2=FH/70.
       FH=F2+TCF
       F2=BL*SV-BD*CV-AD*CV

***    VELOCITY ANGLE AND MAGNITUDE SQUARED
       VAN=ATAN2(Z1,X1)
       V2=Z1**2+X1**2
       SV=SIN(VAN)
       CV=COS(VAN)

***    ANGULAR MOMENT, ACCELERATION, VELOCITY, AND POSITION ABOUT Y AXIS

       A1=INTGRL(A1I,Y1)
       Y1=INTGRL(Y1I,Y2)
       Y2=YM/975.
       YM=YCM+YADC-YADS

***    FIN SERVO LOOP

       FA=INTGRL(FAI,FB)
       FB=INTGRL(FBI,FC)
       FC=FD*1000.
       FD=-.0443*FB-FA+DE+DR+YPR

***    COMPUTATION OF ANGLES OF ATTACK RELATIVE TO THE VELOCITY VECTOR
       AR=A1+VAN
       A=AR*57.29578
       BR=FA+VAN
       B=BR*57.29578
```

```
***    CALLING THE BODY COEFFICIENTS OUT OF THE SUBROUTINE
       CL,CD,CM=TINK(A,B)

***    NEW TENSION FORCE COMPUTATION

       SFS=SIN(FSS)
       LO=ZO/SFS
       TFS=TAN(FSS)
       XO=ZO/TFS
       ZN=ZO-Z
       XR=X1I-X1
       X=INTGRL(XI,XR)
       XN=XO+X
       LN=SQRT(ZN**2+XN**2)
       L=LN-LO
       T=1940.+L*SC

***    CALCULATION OF THE ANGLE FE
       FE=ATAN2(ZN,XN)
       SFE=SIN(FE)
       CFE=COS(FE)
***    HORIZONTAL AND VERTICAL COMPONENTS OF TENSION
       TCF=T*CFE
       TSF=T*SFE
***    COMPUTING ANTENNA DRAG
       ZD=4.+Z
       AD=ZD*.04146*V2
***    COMPUTING BODY LIFT, DRAG, AND MOMENT
       BL=CL*14.17875*V2
       BD=CD*14.17875*V2
       YCM=CM*85.0725*V2
***    COMPUTING MOMENTS CAUSED BY THE ANTENNA
       YADS=AD*SV*3.75
       YADC=(ZD*.5+1.2873)*AD*CV
***    COMPUTING THE THREE INPUTS INTO THE FIN SERVO LOOP
       DEE=.443*Z
       DE=DEE*ADE
       DRR=.443*Z1
       DR=DRR*ADP
       YPR=Y1*APR
***    PROGRAM CONTROL CARDS
CONTRL  FINTIM=30.
PRINT  .1,Z,Z1,FV,X1,FH,T,A,B
END
STOP

***    NOTE--BLANK CARDS AND CARDS MARKED ***, ARE NOT INCLUDED IN ACTUAL PROGRAM
```

FORTRAN

```
      SUBROUTINE TINK(A,B,CL,CD,CM)
      REAL*8 A,B,CL,CD,CM,C1,C2
      CL=.0051D 00*B+2.1712552D-01+2.0279223D-02*A+3.1636558D-04*A**2+2.
     1093616D-05*A**3-3.2986561D-07*A**4-8.1116178D-08*A**5
      CD=4.2707372D-02+3.6589646D-03*A+2.5607579D-04*A**2+1.1961132D-05*
     1A**3+2.9840907D-07*A**4-7.5139831D-08*A**5+3.8575977D-09*A**6+2.57
     203807D-10*A**7-2.323927D-11*A**8+(6.8143195D-04+6.6382999D-05*A+1.
     33323814D-06*A**2+1.5116013D-06*A**3-2.1755085D-08*A**4-2.1159367D-
     408*A**5-1.7507392D-12*A**6+8.1546554D-11*A**7+6.089921D-13*A**8)*B
     5+(4.7428856D-05-3.1706617D-06*A+5.6700361D-07*A**2+1.9501584D-07*A
     6**3-1.6379961D-08*A**4-2.600752D-09*A**5+1.8132173D-10*A**6+9.7843
     775D-12*A**7-6.4075507D-13*A**8)*B**2
      CM=C1+C2
      C1=3.1254762D-02-2.7515873D-03*B-4.8952381D-05*B**2-4.6222222D-06*
     1B**3+(-7.2748931D-03+6.9793037D-06*B+4.9010127D-06*B**2-4.9364676D
     2-07*B**3-8.0377067D-08*B**4)*A+(-6.7932139D-05+5.4210736D-06*B+1.0
     37897310-06*B**2-9.6995988D-08*B**3-4.6666717D-08*B**4-7.2678165D-0
     49*B**5-4.3351393D-10*B**6)*A**2+(-1.2808628D-05-1.6744241D-06*B+2.
     59142678D-07*B**2+2.7469724D-08*B**3-1.4170085D-08*B**4+5.4319291D-
     610*B**5+1.7953234D-10*B**6)*A**3
      C2=(-9.4307248D-07-2.1777229D-08*B+5.4412650D-09*B**2+4.3324391D-0
     19*B**3+4.8010971D-10*B**4-4.7920116D-11*B**5-5.4138508D-12*B**6)*A
     2**4+(3.4636440D-07+3.2453934D-08*B-1.1466209D-08*B**2-1.3877497D-0
     39*B**3+3.9658498D-10*B**4+2.3337991D-11*B**5-1.6875364D-12*B**6)*A
     4**5+(-1.6765721D-08-1.863330D-09*B+7.0670144D-10*B**2+7.5954549D-1
     51*B**3-2.5910672D-11*B**4-1.5201129D-12*B**5+1.0079619D-13*B**6)*A
     6**6
      RETURN
      END
```

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13. ABSTRACT

A two-dimensional dynamic model of a towed buoy designed to remain submerged just below the water surface is developed. The study includes the model development, the analysis of certain model parameters, the development of a tension force, and the range of stability of the model to initial condition perturbations.

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